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## SYNTHESIS OF SHAPE MORPHING COMPLIANT MECHANISMS USING LOAD PATH REPRESENTATION

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### ABSTRACT

Synthesis of shape morphing compliant mechanism is inherently different from typical single output problems, due to the multiple output points along the morphing boundary. Two synthesis approaches, using a fixed initial discretization mesh and using a load path representation, have been developed previously to simultaneously address the topology, size, and geometry aspects of a compliant mechanism. Due to insufficient diversity in later generations, pre-matured convergence to sub-optimal solution is observed in the load path approach. In this paper, additional genetic operation strategies are introduced to enhance the design diversity in each Genetic Algorithm population. The capabilities and limitations of the fixed mesh approach and the improved load path approach are studied through several examples. The results show that the load path approach can achieve better quality solution with less computation time than the fixed mesh approach. The load path representation can potentially lead to a fully systematic synthesis approach due to the absence of a pre-specified initial discretization mesh. Boundary conditions are currently being incorporated into the design variable to understand their importance in structural topology.

**Keywords:** compliant mechanism, shape morphing, load path representation, genetic algorithm, adaptive shape change.

### INTRODUCTION

A compliant mechanism is a single-piece flexible structure that is designed to be flexible to transmit motions, yet stiff enough to withstand external loads. Various synthesis approaches, utilizing structural optimization techniques, have been developed in the past decade to design practical devices in many scales, ranging from motion amplifiers [1] in MEMS to compliant bicycle derailleurs [2]. Another feature of compliant mechanisms is its smooth deformation field, due to the

distributed compliance, that provides a novel concept in shape morphing applications. However, unlike the single output problems studied in most of the previous research, shape morphing involves deforming the morphing boundary from its initial profile to a desired target shape, where every point along the boundary is an output point. Therefore, the shape morphing problem has multiple output points, and it might be inappropriate to use the synthesis approaches tailored for single output problems [3].

In a previous paper [3], we developed a synthesis approach for shape morphing compliant mechanisms, using a Genetic Algorithm (GA) to simultaneously optimize the structural topology and dimensions. The approach starts with a preprocessor to estimate if the shape morphing is achievable by studying the curve length and curvature of both the initial and target curves. The preprocessor also identifies several candidate points along the morphing boundary to be the 'output points' of the compliant mechanism. The synthesis approach then discretizes the design domain into a finite element (FE) network (mesh) using beam elements. Each beam element is described by two design variables: one binary variable determining the presence of an element (topology), and one real variable defining the cross section area (dimension). This will be referred to as the 'fixed mesh approach' in this paper. Although the results have demonstrated the feasibility of simultaneously addressing the topology and dimensional aspects of compliant mechanisms, it is still unclear how the initial discretization network (configuration and complexity) is selected. Furthermore, since the design variable definition can actually lead to invalid designs, such as structures that are not grounded, an additional algorithm has to be used to check the structural connectivity and penalize the disconnected designs. However, the structural connectivity is not explicitly expressed in terms of the design variable, so the checking algorithm has to

'search for' the connectivity information, leading to inefficiency in the overall GA process. Moreover, due to the direct correspondence between the design variables and the elements, the number of design variables will increase dramatically when a more refined initial mesh is used, thus increasing the solution space that GA needs to search within.

To address the issues arising from the use of an initial discretization mesh, we later developed a new design domain parameterization method, utilizing the load paths as the design variables [4]. This parameterization scheme is incorporated into a synthesis procedure identical to that in the fixed mesh approach (including the same preprocessor), but the fixed FE mesh parameterization is replaced by the load path representation. This will be referred to as the 'load path approach' in this paper. The load paths are the physical connections that deliver energy between the input actuator and the output points. Using the load paths representation allows the structural connectivity information to be included directly in the design variables, thus eliminating the need of an additional checking algorithm. Moreover, the load path representation utilizes a set of intermediate connection ports to control the connectivity between different load paths; therefore, topologies with various configuration and complexity can be generated without specifying an initial discretization element network. Because the design variables are defined from the structural connectivity point of view instead of in the *element* level, the number of design variables in the load path representation is, thus, independent of the number of elements used in the structural analysis. Although the results have shown that this new representation method can effectively exclude the invalid designs from the solution space and create well connected structures, the GA populations in later generations seemed to contain many copies of the same design (dominant solution). This indicates that the genetic operations used in the reproduction process can not maintain sufficient diversity in each generation, leading to pre-matured convergence to the dominant solution.

In this research, we focus on the optimization process in the load path approach and introduce additional genetic operations within the reproduction scheme to increase the design diversity in each generation. Two additional searching algorithms are also employed following the GA to further explore the solution space and accelerate the convergence to a local (or global) optimum. This improved load path approach is applied to several examples to study its advantages over the fixed mesh approach. A brief overview of the load path representation will be introduced in the next section, followed by the discussion on the enhancements in GA and the local search methods.

### LOAD PATH REPRESENTATION METHOD

In every compliant mechanism, three essential categories of points must exist to ensure proper functionality of the system: (1) at least one input point to connect to the actuator; (2) at least one fixed point to support and ground the structure; and (3) one or more output points (obtained from the preprocessor [3]) to interact with the external environment. As can be seen in Fig. 1, these essential points should be connected

directly or indirectly to each other to form a valid compliant mechanism. It is assumed that each essential point is connected to every other essential point in different categories only, and the connection between every pair of essential points (in different categories) is represented as one load path. Thus, three types of load paths will be generated: (1) paths from Input  $\rightarrow$  Output, (2) paths from Input  $\rightarrow$  Fixed points, and (3) paths from Fixed point  $\rightarrow$  Output. In addition, some intermediate connection ports are introduced to allow connections between different types of paths to achieve 'indirect' connections between input/output/Fixed points. Based on this layout, each design can form a graph with vertices consist of the input, output, fixed points, and intermediate connection ports, as shown in Fig. 2. The paths in Fig. 2 are then randomly generated by GA in terms of the vertices as the 'path sequences' listed in Table 1. For each load path, there is also a corresponding binary variable,  $p_{Top}$ , which indicates the presence or elimination of this path: when all  $p_{Top} = 1$ , the graph is considered 'fully connected;' when some  $p_{Top} = 0$  in a graph, the  $p_{Top} = 0$  paths are eliminated from the design. For example, the design in Fig. 3 has the same path sequences as Fig. 2, but different  $p_{Top}$  values create a different topology. The  $p_{Top}$  values are shown in Table 1 (5<sup>th</sup> column), while the path sequences (3<sup>rd</sup> column) remain the same as those in Fig. 2. The path sequences and  $p_{Top}_i$  are, therefore, considered the topology design variables. To ensure all designs are valid (connected), each design must satisfy two connectivity requirements [4]: (1) the structure needs to be grounded with one or more fixed points, and (2) the input has to be connected to the rest of the structure. These rules can easily be incorporated in the load path approach by monitoring the  $p_{Top}_i$  values of paths from input and fixed point(s) to the output points. At least one of the  $p_{Top}_i$  values in each path type has to be 1. This is a huge advantage over the fixed mesh approach where a topology variable only determines the existence of *one* element, because the structural connectivity is now shown explicitly in the design variables.

## Structural Connectivity

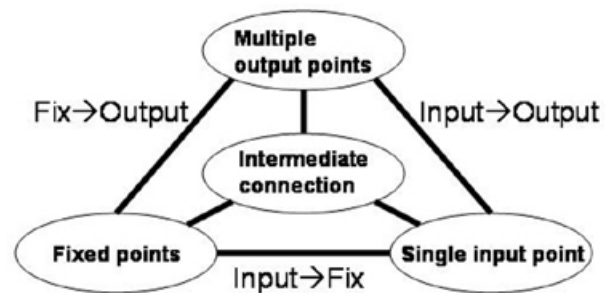


Figure 1: The input point, fixed point, and output points should be connected directly or indirectly by load paths. Note that these paths are non-directional.

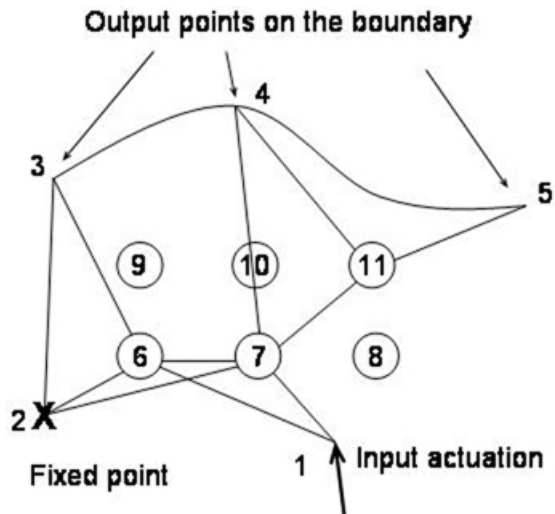


Figure 2: An example design with 6 intermediate connection ports. This design is considered fully connected because  $pTop_i = 1$  for all paths ( $i = 1 \sim 7$ ).

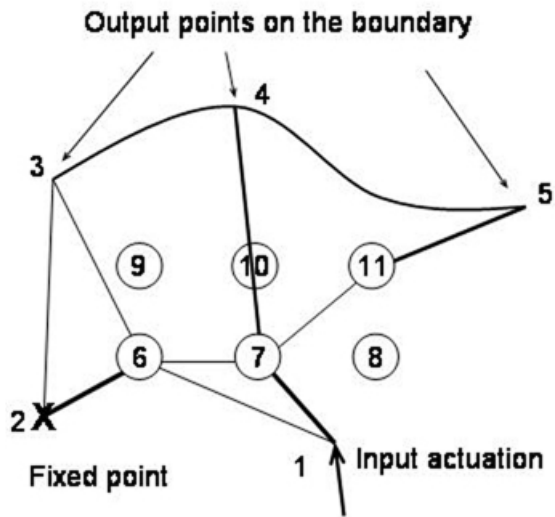


Figure 3: An example design with identical path sequences as in Fig. 2, but different  $pTop_i$  values (5<sup>th</sup> column, Table 1) lead to a different topology.

Table 1: The path sequences and  $pTop$  for the designs in Fig. 2 and Fig. 3. Note that the path sequences are identical for the two designs. The  $pDim$  for Fig. 3 is also included.

Path type	Path No.	Path sequence	$pTop_i$ Fig.2	$pTop_i$ Fig.3	$pDim_i$ Fig.3
In ↓ Out	1	{1,6,3}	1	1	{1,0.75}
	2	{1,7,11,4}	1	0	{1,1,2}
	3	{1,7,11,5}	1	1	{3,0.75,2.5}
In ↓ Fix	4	{1,6,2}	1	1	{2.5,1.3}
	5	{2,3}	1	1	{0.75}
Fix ↓ Out	6	{2,6,7,4}	1	1	{3,1.5,2.25}
	7	{2,7,11,5}	1	0	{1.8,1.1}

It is assumed that the load paths are comprised of beam elements, thus the size design variables are then defined as the cross section dimensions of each beam section between two vertices. Take Fig. 3 for example, each path has a corresponding sequence,  $pDim_i$ , to describe the section dimensions, shown in Table 1. When two load paths have overlapping sections, such as in path #1 and #4 between vertices 1 and 6 (Table 1), one of the two available  $pDim$  values (1 and 2.5 from  $pDim_1$  and  $pDim_4$ ) is randomly selected to describe the section dimension. Since only one value is required to define one section, this random selection can be seen as the selected value dominating the recessive one. An additional variable,  $hBoundary$ , is also included in each design to define the cross section dimension of the morphing boundary. Finally, to control the geometry of the compliant mechanism, the geometry design variables are defined as the locations of the intermediate connection ports. Using the design variables defined in the load path representation, the topology, size, and geometry of the compliant mechanism can be optimized simultaneously in the GA synthesis approach. Interested readers can refer to Lu and Kota [4] for more details regarding the load path approach.

Although the load path approach can effectively exclude invalid designs from the solution space, the solution in each generation in GA still improves very slowly. By studying the composition in each generation, we found that later generations tend to have multiple copies of the same design (the dominant design). Sometimes more than 80 percent of the designs in one generation are identical. This indicates that the genetic operation schemes used in the load path approach were unable to maintain sufficient diversity in each generation, thus leading to pre-matured convergence to the dominant design. In the next section, we will study the genetic operation schemes used in the load path approach [4] and introduce additional crossover and mutation strategies to increase the diversity in each generation.

#### MAINTAINING DIVERSITY IN GENETIC ALGORITHM

GA simulates the selection scheme found in nature and evolves a population based on the principle of 'survival of the fittest' in order to find the optimal design that performs best at a required task (the objective function). In the shape morphing problem, the goal is to morph the structural boundary from its initial profile to a target shape. The structural deformation is solved for using a Finite Element Analysis (FEA), and the deformed boundary is then compared to the desired target shape to evaluate the performance (fitness) of a design. The smaller the deformed shape deviates from the target shape, the better the shape morphing is achieved, hence the fitter the design is. In the GA, the designs in the first generation are randomly generated to somewhat sample the whole solution space. This parent generation then produces a new generation from a reproduction scheme which includes selection, crossover, and mutation. The load path approach incorporates a roulette wheel selection scheme, where fitter individuals have higher chances to be selected for reproduction, and inferior ones have lower probability to reproduce. The selected parent designs then produce new offspring designs through the genetic operations: crossover and mutation. The genetic operations create diversity within each generation, which, in fact, provides the power

behind GA to improve designs as generations evolve. The crossover and mutation strategies are, therefore, essential to the performance of GA.

In the load path approach, it is observed that later generations tend to contain multiple copies of the same design. This implies that the genetic operations cannot maintain sufficient diversity in each generation. In the load path approach, the crossover strategy is to ‘exchange’ the path sequence, pTop, and pDim information of several randomly selected paths between two parent designs. However, when the two parent designs are identical, the ‘exchanging paths’ strategy fails to produce any new design. In fact, the offspring designs will still be the same as the two identical parent designs. This means the main source to generate new designs comes only from the mutation operation.

Possible ways to enhance the diversity in each generation include increasing the mutation probability and incorporating additional mutation strategies. The mutation strategies used in the load path approach include mutating the boundary dimension (hBoundary) and changing the path destination (last vertex in path sequence) of a randomly selected path [4, 5]. However, the intermediate connection port locations have remained unchanged throughout the whole genetic operations. Therefore, an additional mutation strategy is introduced by mutating the location of a randomly selected connection port, varying the element geometry in the compliant mechanism. To further increase the diversity, the binary topology variable (pTop) is also allowed to mutate from 0 to 1 and vice versa for one randomly selected load path, thus changing the topology. These additional mutation strategies help preserve necessary diversity within each generation. They can also enhance the crossover performance, because the more diverse a generation is, the less likely it is to select two identical parent designs. Thus, the crossover strategy (exchanging path information) can function properly and produce new offspring designs.

#### CONVERGENCE TO GLOBAL/LOCAL OPTIMUM

Due to the heuristic nature of GA, the algorithm is capable of searching the whole solution space more extensively without being trapped in a local region. Although GA is more efficient in locating a region *close to* a local optimum, finding the *exact location* may be quite difficult. If the GA can indeed explore the entire solution space thoroughly, performing a local search following the GA can accelerate the convergence to the nearest local optimum, which is very likely to be the global optimum. However, there is no guarantee that GA can explore or *sample* the solution space evenly, so adding a local search after GA can only lead to a local optimum. In order to enhance the chance of finding the global optimum, a global search, DIRECT optimization algorithm [5], is adopted to help investigate the global optimality. DIRECT optimization algorithm is a sampling algorithm that requires no knowledge of the objective function gradient. The algorithm samples points in the solution space and uses the information it has obtained to decide where to search next. It operates at both the global and local level. Once the global part of the algorithm finds the basin of convergence of the optimum, the local part of the algorithm quickly and automatically exploits it [5]. We, therefore, take

the optimal solution obtained from GA and use its topology as a basic layout for the DIRECT algorithm to perform additional iterations on the connection port locations and beam section dimensions. As shown later in the results, the DIRECT algorithm can effectively improve the design performance with the same topology. However, the sampling nature of DIRECT algorithm implies that the obtained solution depends greatly on the number of iterations (sampling points). Therefore, a local search algorithm is utilized to accelerate the convergence to the nearby local optimum, after a prescribed number of iterations are carried out using the DIRECT algorithm. In this paper, the optimization toolbox in Matlab is used to perform the local search. More information on the optimization toolbox can be found in the Matlab documentation. The improved load path approach using GA, global search, and local search, is shown in Fig. 4.

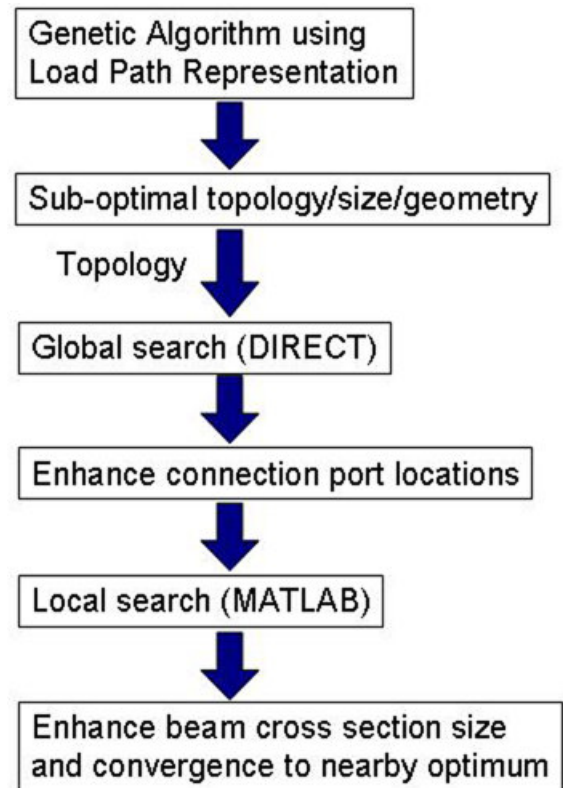


Figure 4: The improved load path approach using the synthesis approach in GA followed by a global search and a local search to improve the convergence to a local optimum.

#### EXAMPLES AND DISCUSSIONS

Shape morphing can be seen useful in many areas, such as changing the aircraft wing shape to reduce drag, or changing the lumbar support shape in chairs to enhance comfort. To study the performance of the synthesis approach for compliant mechanisms, the fixed mesh approach and the improved load path approach are applied to several examples in the following to understand their capabilities and limitations. All of the examples use the Least Square Error (LSE) deviation shown in Eq. (1) as the objective function in GA to evaluate the difference between the achieved shape (deformed curve) and the desired target curve, subject to size, node locations, stress, and stiffness constraints. Interested readers can refer to our

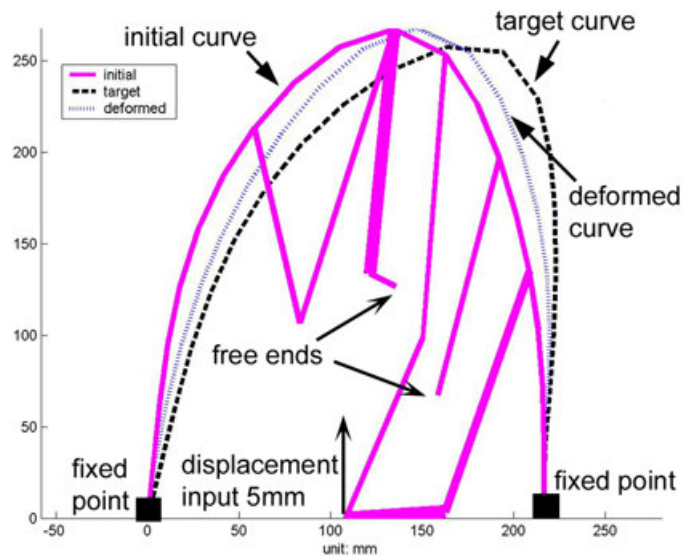
previous paper [4] for more details regarding the problem formulation.

$$LSE_{dev} = \frac{1}{n} \sum_{i=1}^n \sqrt{(x_{DEF,i} - x_{TAR,i})^2 + (y_{DEF,i} - y_{TAR,i})^2} \quad (1)$$

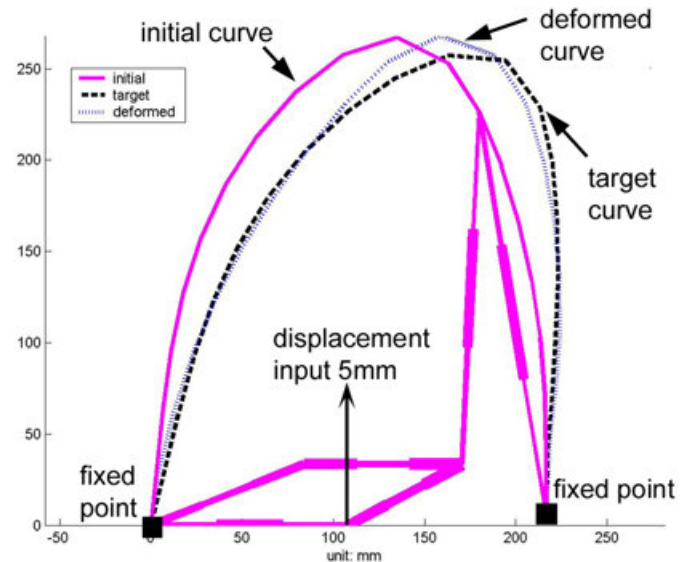
where  $n$  is the total number of data points along the morphing boundary;  $(x_{DEF,i}, y_{DEF,i})$  and  $(x_{TAR,i}, y_{TAR,i})$  are the  $i^{th}$  data point on the deformed and target morphing boundary.

### Morphing Aircraft Leading Edge

Most aircraft wings are optimized to produce minimum drag under a particular flying speed, at which the largest proportion of fuel is expended. However, in reality, flying speed varies continuously throughout flight. Hence, to obtain optimal fuel efficiency, the wing shape should be able to change in response to the change in flying speed [6]. The shape morphing of a hypothetical airfoil leading edge is investigated here to compare the performance of the fixed mesh approach and load path approach. The optimal structure obtained from the fixed mesh approach is shown in solid lines in Fig. 5 with an LSE deviation of 8.92mm (0.35inch). The dark dash line shows the target shape and the light dash line is the actual boundary shape (deformed curve) after structural deformation due to input actuation. The optimal solution obtained from the load path approach is shown in Fig. 6 with an LSE deviation of 3.72mm (0.15inch). Both solutions are obtained using the same number of population (150), number of generation (50), crossover probability (0.8), and mutation probability (0.5). In addition, the output points along the morphing boundary are determined in the preprocessor [3]. The overall dimension is 230mm (9.06inch) by 260mm (10.24inch) by 20mm (0.79inch) (out-of-plane), and the material is aluminum.



**Figure 5: The optimal compliant mechanism obtained from ten trials of the fixed mesh approach for airfoil leading edge shape morphing. Solid line: structure topology in initial (undeformed) shape; dark dash line: target curve; light dash line: the achieved shape after deformation due to input actuation.**



**Figure 6: The optimal compliant mechanism obtained from ten trial runs of the load path approach for airfoil leading edge shape morphing.**

**Table 2: The LSE deviation value and computation time from ten trials of both approaches.**

Aircraft Leading Edge Example	Fixed Mesh Approach	Load Path Approach
LSE dev. of best design	8.9193mm	3.7189mm
Average LSE dev.	9.4540mm	6.7385mm
Average CPU time	533sec (8.88min)	274sec (4.57min)

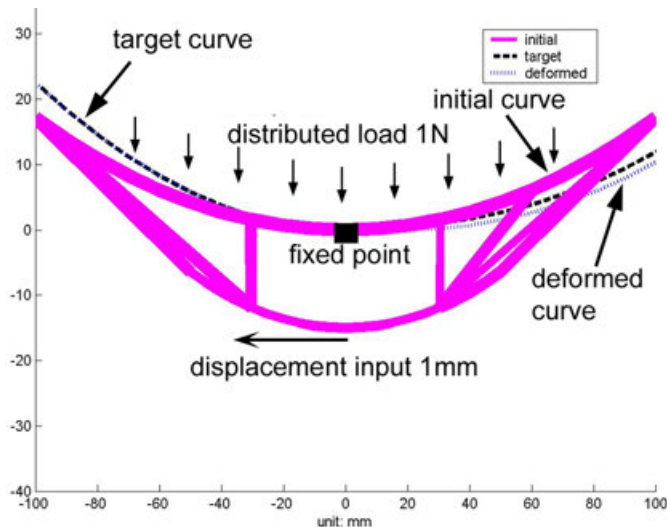
Due to the heuristic nature, GA can provide a different result for the same problem in each run. The designs shown in Fig. 5 and Fig. 6 are the best solutions after performing ten trial runs of each approach. The average computation time and LSE deviation for the ten trials are shown in Table 2. As can be seen, the fixed mesh approach requires almost twice the computation time of the load path approach. This may result from the larger number of design variables used in the fixed mesh approach. Moreover, the checking algorithm for structural connectivity may also lead to excessive computation time. Since the mesh is fixed in this approach, the optimal solution is always a subset of all possible designs embedded in the initial discretization mesh. However, the *true* optimal solution might not be included in the initial mesh. Therefore, the selection of the initial mesh is critical to the quality of the final solution. Note that there are several ‘trivial’ elements that have one ‘free end’ as shown in Fig. 5. These elements have no strain/stress in them, so they can be removed without affecting the compliant mechanism performance.

The load path approach, on the other hand, can generate various structural topologies, because the locations of the connection ports are part of the design variables. The use of load path representation also eliminates the need of an additional checking algorithm for connectivity. Therefore, the computation time is reduced and the desired shape morphing can be achieved with smaller deviation. As seen in Fig. 6, all elements are connected at both ends because the topology is

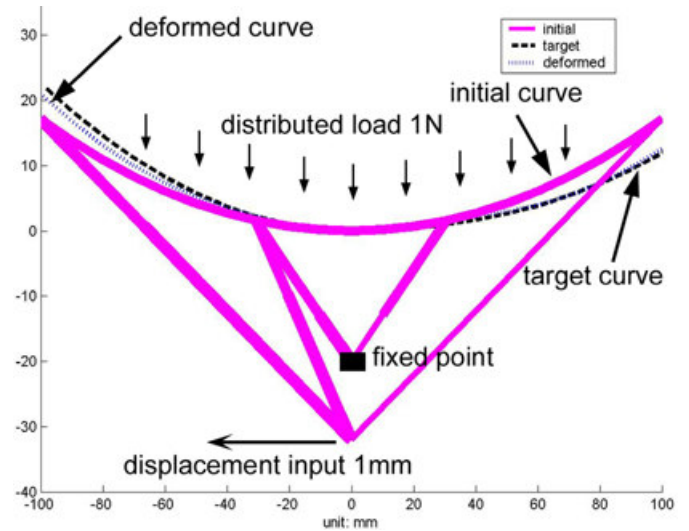
now represented in terms of the load path, thus there are no more trivial elements.

### Flexible Antenna Reflector

Recent studies [7-10] have shown that antenna reflector adaptation can potentially enhance system performance and increase flexibility, such as changing the signal pattern or coverage area. In this example, a flexible antenna reflector changes its shape to direct the radiation signal to a different direction. As shown in Fig. 7 and Fig. 8, the two tips of the cylindrical reflector move in opposite directions to simulate a rotation about the center. Figures 7 and 8 are the optimal solutions from the fixed mesh approach and the load path approach respectively with the corresponding LSE deviation values of 0.53mm (0.02inch) and 0.51mm (0.02inch). The overall dimension of the reflector is approximately 200mm (7.87inch) by 40mm (1.57inch) by 4mm (0.16inch) (out-of-plane) and the material is ABS (Acrylonitrile-Butadiene-Styrene) plastic. As can be seen, both designs are able to achieve the desired shape morphing of less than 2% of the shorter overall dimension. Note that both designs were obtained from only one trial run of each algorithm, while multiple trials are sometimes necessary for more complicated shape morphing. This suggests that the fixed mesh approach and load path approach are equally effective in finding a design that can achieve the desired shape morphing when the problem is less complex. As we will describe later, the complexity is typically related to the number of inflection points in the problem.



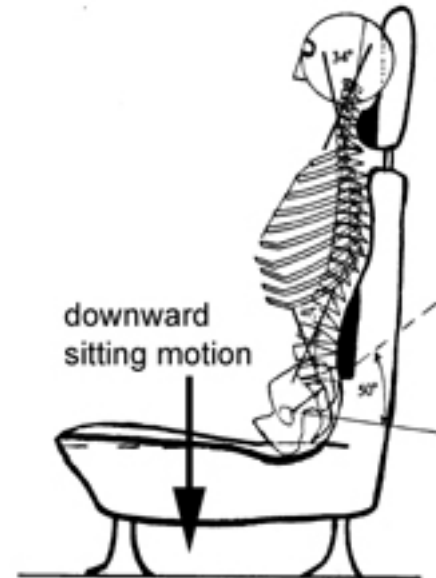
**Figure 7: The optimal solution for antenna reflector beam steering obtained from the fixed mesh approach.**



**Figure 8: The optimal solution for antenna reflector beam steering obtained from the load path approach.**

### Shape Morphing Lumbar Support

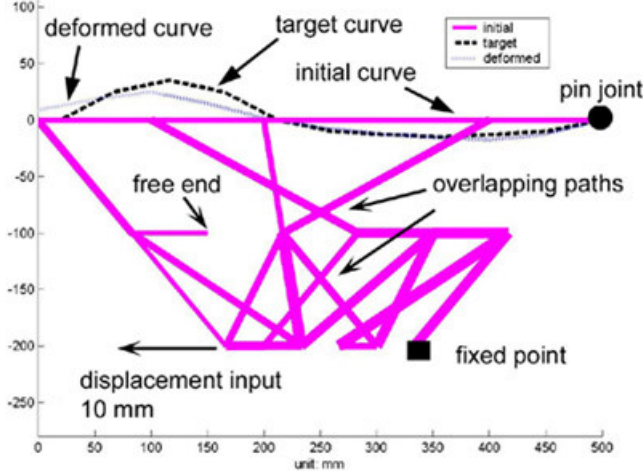
Lower back pain occurs frequently and is one of the most costly health problems affecting industry and society. Lifetime prevalences of 60% to 90% have been reported [11]. Lumbar support is one of the commonly used preventive strategies [12]. This example is inspired by the lumbar support system that is commonly used in car seats and office chairs to prevent lower back pain. The downward ‘sitting’ motion from a person is used as an input to actuate the lumbar support that changes the initially straight back support shape into a curved profile. The curved profile should match the natural profile of human spine, which typically includes an inflection point as shown in Fig. 9.



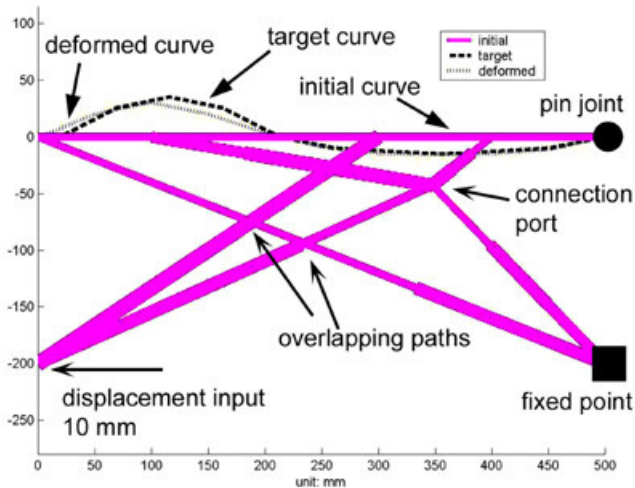
**Figure 9: Natural sitting spinal model in an ideal driver’s seat [13].**

To study the performance of the fixed mesh and load path approaches, we created a curve to roughly approximate the spinal shape. The initial (straight) and target (spine) curve are shown horizontally in Fig. 10 and Fig. 11, thus the originally ‘downward’ input motion becomes an input to the left. Figures

10 and 11 are the best designs obtained from 10 trial runs of the fixed mesh approach and load path approach each. The corresponding LSE deviation values are 11.24mm (0.44inch) and 10.55mm (0.42inch) respectively. The overall dimension is 500mm (19.69inch) by 200mm (7.87inch) by 5mm (0.2inch) (out-of-plane) and the material is ABS plastic. The average LSE deviation and required computation time are listed in Table 3.



**Figure 10: The optimal lumbar support design obtained from the fixed mesh approach.**



**Figure 11: The optimal lumbar support design obtained from the load path approach. Note that relative motions are allowed between overlapping paths, while all paths are joined together (no relative motion) at the connection ports.**

**Table 3: The LSE deviation value and computation time from ten trials of both approaches.**

Lumbar Support Example	Fixed Mesh Approach	Load Path Approach
LSE dev. of best design	11.2738mm	10.5463mm
Average LSE dev.	12.3716mm	10.6284mm
Average CPU time	460sec (7.67min)	243sec (4.05min)

As can be seen in Table 3, the fixed mesh approach requires almost twice as much computation time as the load path approach. Although the LSE deviation values are close in

Fig. 10 and Fig. 11, the shape morphing does look different (visually). This may result from the insufficient numbers of data points along the initial and target curves. There are currently 11 data points along the curve, and the Euclidian distance between the deformed and target location of each point are used to measure the LSE deviation in Eq. (1). Additional data points along the morphing boundary can potentially lead to more significant difference between the LSE deviations obtained from the two approaches.

Simple cantilever beam bending can be seen as the whole beam pivots about the fixed end. However, creating an inflection point requires changing the center of curvature from one side to the opposite side of the beam. Since it requires a moment or some opposite (push/pull) motions to generate a couple at the inflection point, shape morphing involving creating an inflection point, such as this lumbar support example, is considered more complicated than simple cantilever beam bending (antenna reflector example). It is observed that, from the 10 trial runs using fixed mesh approach, the resulting designs can typically achieve the shape morphing on the right hand side (flatter side) in Fig. 10, but most of them fail to match the portion on the left of the inflection point. In other words, it is relatively straightforward for the algorithm to find a solution without an inflection point, but generating an inflection point can be quite difficult. On the other hand, the load path approach appears to capture the inflection point better. As can be seen in Fig. 11, the overlapping paths from the input and fixed point to the opposite sides of the inflection point seem to provide a push/pull motion, leading to a moment about the inflection point. This overlapping 'X' configuration can also be vaguely seen in the result in Fig. 10. However, if this X configuration is not included in the initial mesh, the fixed mesh approach may not be able to generate an inflection point at all. We included the X configuration in the initial mesh based on our understanding of the problem and some results obtained from the load path approach, but this kind of information is generally unavailable for other problems. Therefore, we believe the load path approach is a better means to systematically design shape morphing compliant mechanisms without the need of intuition or prior experience to define an initial discretization element network.

**FINAL REMARKS**

In this paper, we introduced the load path representation method and the enhancements in GA to increase design diversity. Several examples are included to study the performance of the load path approach and the fixed mesh approach. The results showed that the load path approach requires almost half of the computation time required by the fixed mesh approach, due to fewer design variables and the absence of additional checking algorithm for structural connectivity. Generally, both approaches are able to achieve simple shape morphing when no inflection points are involved. However, for more complicated problems with inflection points, load path approach can provide solutions that are able to improve our understanding to a problem, such as the required topology to generate a particular motion. More importantly, unlike the fixed mesh approach, the load path approach does not have an initial discretization network which typically requires intuition or prior experience when determining the

complexity and configuration of the mesh. Therefore, the load path approach can potentially lead to a fully systematic synthesis approach for shape morphing. Although we are constantly looking for more application areas where shape morphing compliant mechanisms can be useful, the load path representation is not limited to multiple output problems. For single output problems, the load path representation can be used simply by replacing the curve comparison objective function with the objective function used in the single output system. In other words, the load path representation is a general method to parameterize the design domain into appropriate design variables to simultaneously optimize the compliant mechanism topology and dimensions. Future and on-going research includes the study of boundary conditions since the locations of the input actuator and ground supports also play significant roles in the structural topology. Therefore, the boundary conditions should be included in the design variables to further explore the optimal structural topology.

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