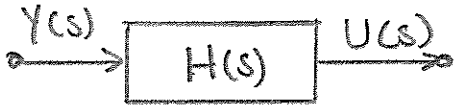
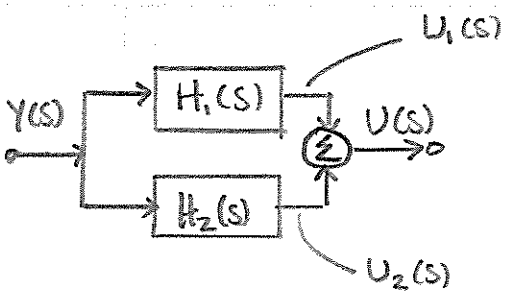
CLASS 15 - BLOCK ANALYSIS & STEP ANALYSIS

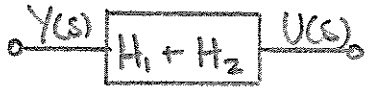


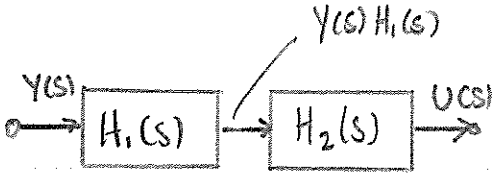
$$U(s) = H(s)\,Y(s)$$



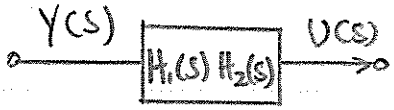
$$U_1(s) = H_1(s)\,Y(s)$$
$$U_2(s) = H_2(s)\,Y(s)$$

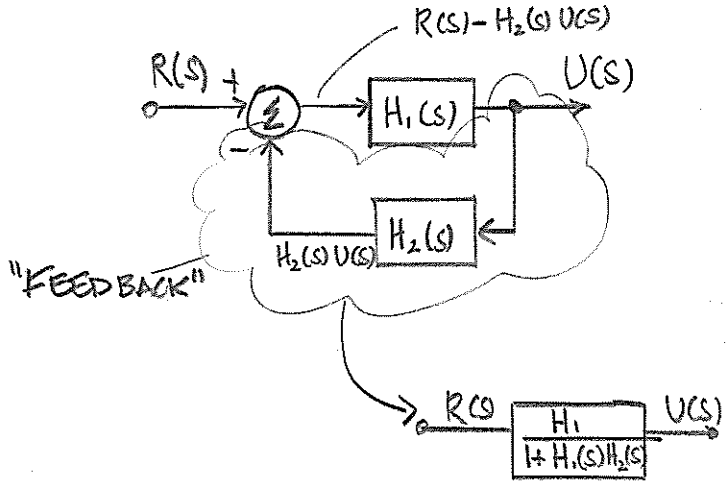
$$U(s) = U_1(s) + U_2(s)$$
$$= \left[H_1(s) + H_2(s)\right] Y(s)$$





$$U(s) = H_2(s)\,H_1(s)\,Y(s)$$



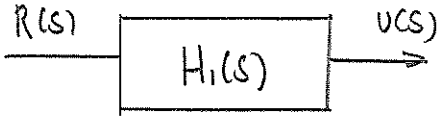


$$U(s) = H_1(s)\left[R(s) - H_2(s)\,U(s)\right]$$

$$U(s) + H_1(s)\,H_2(s)\,U(s) = H_1(s)\,R(s)$$

$$U(s)\left[1 + H_1(s)\,H_2(s)\right] = H_1(s)\,R(s)$$

$$\frac{U(s)}{R(s)} = \frac{H_1(s)}{1 + H_1(s)\,H_2(s)}$$



"OPEN LOOP"

CONSIDER STEP INPUT, $R(s) = \frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} r(t) = 1(t)$

$$U(s) = \frac{1}{s} H_1(s)$$

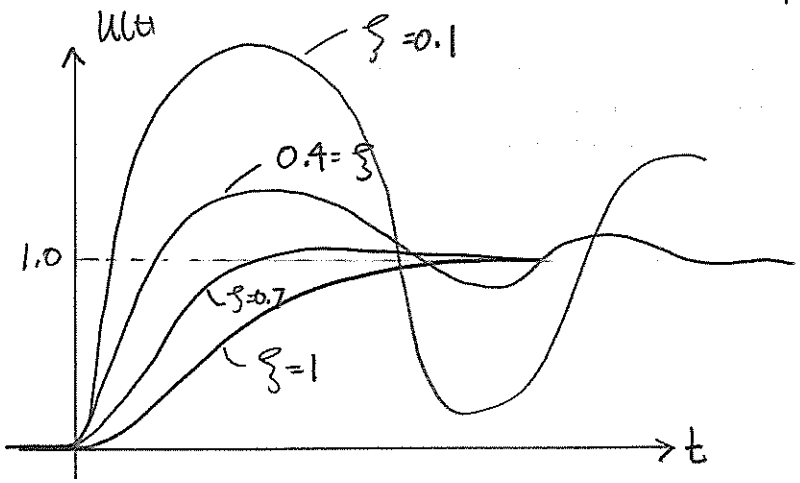
$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

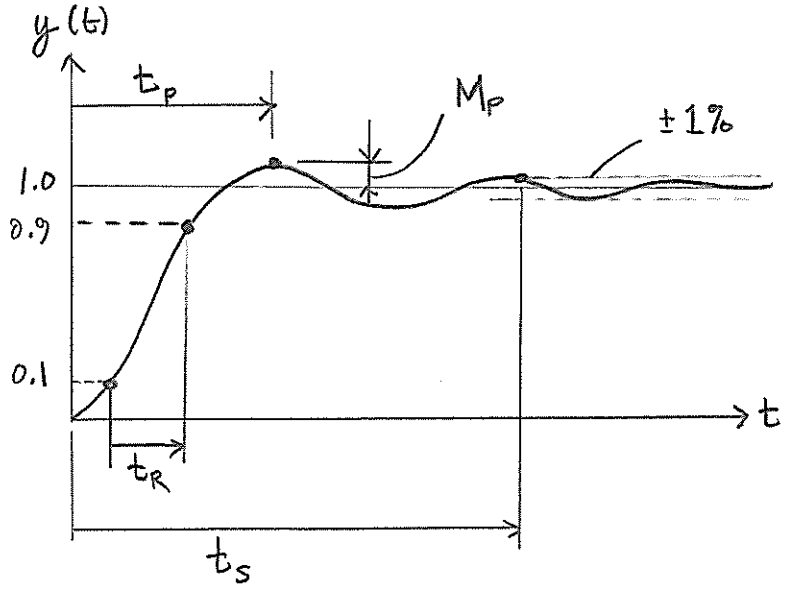
$$U(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s}$$

$$= \frac{(\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}{s\left[(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)\right]}$$

(LAPLACE TABLE #21)

$$u(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\left(\omega_n\sqrt{1-\zeta^2}\,t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right)\right)$$





t_R = RISE TIME (SYSTEM RESPONSIVENESS)

t_s = SETTLING TIME (PRESENCE of TRANSIENTS)

t_p = PEAK TIME

M_p = OVERSHOOT

$$t_R \cong \frac{1.8}{\omega_n}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad (0 \le \zeta < 1)$$

$$t_s = \frac{4.6}{\sigma}$$

SAY WE WANT STRUCTURE WITH STEP RESPONSE of:

$t_R \leq 0.6$ sec , $M_p \leq 10\%$, $t_s \leq 3$ sec.

$\frac{1.8}{\omega_n} \leq 0.6$ sec

$\omega_n \geq 3.0$ rad/sec

$e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 0.1$

$\zeta \geq 0.6$

$\frac{4.6}{\sigma} \leq 3$

$\sigma \geq 1.5$ sec