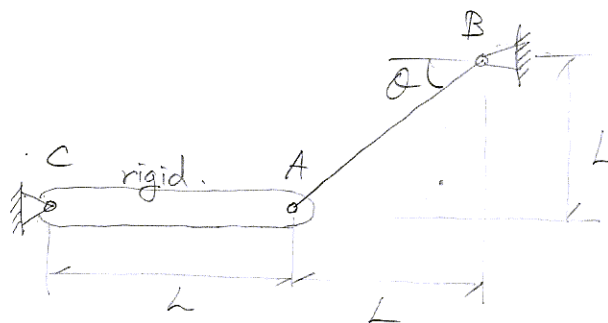


Recitation Class #2.

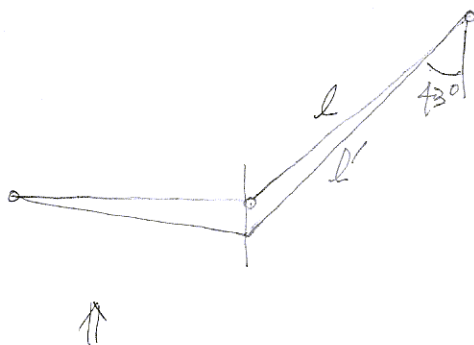
- Rebar tension test.
- 2 Example problems about "Strain"
- Makeup of Class #10.

Example 1 Normal strain. (6th Prob. 2-10, 11).



The wire AB is unstretched when $\theta = 45^\circ$

(a) Determine the normal strain of AB when $\theta = 47^\circ$.



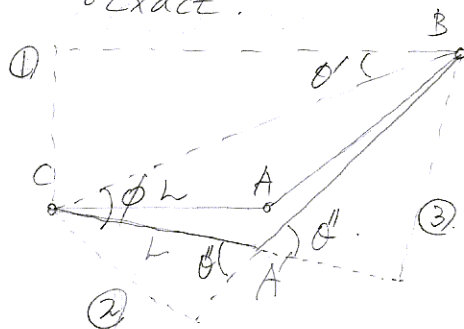
$$L' \sin 43^\circ = L$$

$$L' = \frac{L}{\sin 43^\circ} = 1.466L$$

$$L = \frac{L}{\sin 45^\circ} = 1.414L$$

Assume small deformation. $\epsilon_{AB} = \frac{(1.466 - 1.414)L}{1.414L} = 0.0368$

Exact.



$$CB = \sqrt{(2L)^2 + L^2} = \sqrt{5}L$$

$$\sqrt{5}L \sin \theta' = L$$

$$\sin \theta' = \frac{1}{\sqrt{5}} \Rightarrow \theta' = 26.565^\circ$$

$$\angle ABC = 45 - 26.565 = 18.435^\circ$$

$$\angle A'BC = 18.435 + 2 = 20.435^\circ$$

$$\sqrt{5}L \sin 20.435^\circ = L \sin \theta'' \Rightarrow \theta'' = 51.326^\circ$$

$$\phi + \angle A'BC + \theta'' = 180^\circ \Rightarrow \phi = 180 - 20.435 - (180 - 51.326) = 30.891^\circ$$

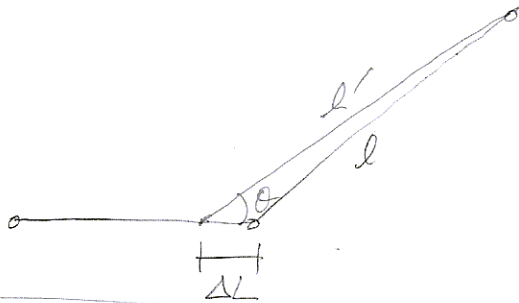
$$\sqrt{5}L \sin \phi = \overline{A'B} \sin \theta'' \Rightarrow \overline{A'B} = \frac{\sqrt{5}L \sin \phi}{\sin \theta''} = 1.470L$$

$$\epsilon_{AB} = \frac{(1.410 - 1.414)L}{1.414L} = 0.0399$$

$$* \text{ Difference : Error} = \frac{0.0368 - 0.0399}{0.0399} = \underline{\underline{-7.15\%}}$$

* When θ is small enough, the error is negligible.

(b) Determine ϵ_{AB} when AC compressed ΔL .



$$l \cos 45^\circ = L \Rightarrow l = \sqrt{2}L$$

$$l' \cos \theta = L + \Delta L$$

$$l' \sin \theta = L$$

$$\cos^2 \theta = \left(\frac{L + \Delta L}{l'} \right)^2$$

$$\sin^2 \theta = \left(\frac{L}{l'} \right)^2$$

• Small deformation.

$$\epsilon_{AB} = \frac{\Delta L \sin 45^\circ}{\sqrt{2}L} = \frac{\Delta L}{2L}$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta = 1 = \frac{(L + \Delta L)^2 + L^2}{(l')^2}$$

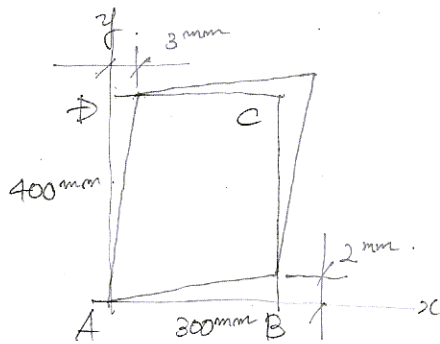
$$\Rightarrow l' = \sqrt{(L + \Delta L)^2 + L^2}$$

$$\epsilon_{AB} = \frac{l' - l}{l} = \frac{\sqrt{(L + \Delta L)^2 + L^2} - \sqrt{2}L}{\sqrt{2}L} = \sqrt{\frac{2L^2 + 2L\Delta L + (\Delta L)^2}{2L^2}} - 1$$

$$= \sqrt{1 + \frac{(\Delta L)}{L} + \frac{1}{2} \left(\frac{\Delta L}{L} \right)^2} - 1 \stackrel{\text{H.O.T.}}{=} \sqrt{1 + \frac{(\Delta L)}{L}} - 1$$

$$\epsilon_{AB} = \sqrt{1 + \frac{(\Delta L)}{L}} - 1 \stackrel{\text{H.O.T.}}{=} 1 + \frac{1}{2} \frac{(\Delta L)}{L} - 1 = \frac{\Delta L}{2L}$$

Example 2 Shear deformation. (6th Prob. 2-25, 2-26)

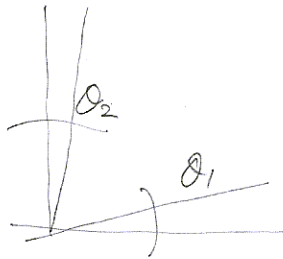


(a) Determine γ_{xy}

(b) Determine ϵ_{DB} , ϵ_{AD}

° Small deformation. $\tan \theta = \sin \theta = \theta$.

(a)



$$\theta_1 = \tan \theta_1 = \frac{2}{300}$$

$$\theta_2 = \tan \theta_2 = \frac{3}{400}$$

$$\begin{aligned} \gamma_{xy} &= \theta_1 + \theta_2 = \frac{2}{300} + \frac{3}{400} = \frac{8+9}{1200} = \frac{17}{1200} \\ &\approx 0.0142 \text{ rad.} \end{aligned}$$

$$\therefore \gamma_{xy} = \frac{17}{1200} = 0.0142 \text{ rad}$$

(b) $DB = \sqrt{300^2 + 400^2} = 500$

$$D'B' = \sqrt{(300-3)^2 + (400-2)^2} = 496.60$$

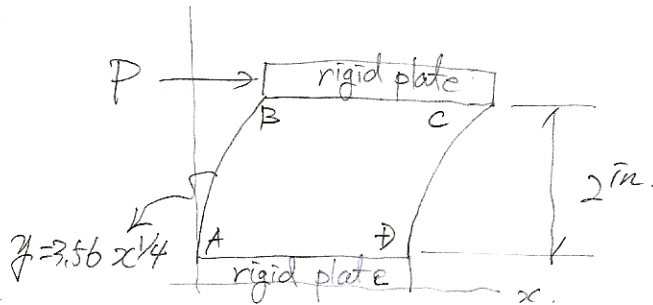
$$\epsilon_{DB} = \frac{496.60 - 500}{500} = -0.006797 \text{ mm/mm} = \boxed{6797 \mu\epsilon}$$

$$AD' = \sqrt{3^2 + 400^2} = 400.011$$

$$\epsilon_{AD} = \frac{400.011 - 400}{400} = 0.00028 \text{ mm/mm} = \boxed{28 \mu\epsilon}$$

Example 3 Shear strain.

A = polystyrene block

Determine γ_A^o & γ_B^o .

$$\frac{dx}{dy} \Big|_{y=y} = \theta = \tan \theta = \frac{dx}{dy} \Rightarrow y = 3.56 x^{1/4}$$

$$y^4 = 3.56^4 x$$

$$x = 0.006226 y^4$$

$$\frac{dx}{dy} = 0.02490 y^3$$

$$\gamma_A^o = \left. \frac{dx}{dy} \right|_{y=0} = 0 \quad \text{rad.}$$

$$\gamma_B^o = \left. \frac{dx}{dy} \right|_{y=2} = 0.199 \quad \text{rad.}$$