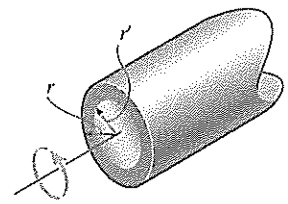
5–3. The solid shaft of radius r is subjected to a torque $\mathbf{T}$. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque $(T/4)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $\tau_{max} = \frac{Tc}{J} = \frac{T(r)}{\frac{\pi}{2}(r^4)} = \frac{2T}{\pi r^3}$

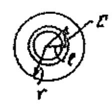
Since $\tau = \frac{r'}{r}\tau_{max} = \frac{2Tr'}{\pi r^4}$

$\tau' = \frac{T'c'}{J'}$; $\quad \frac{2Tr'}{\pi r^4} = \frac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$

$r' = \frac{r}{4^{\frac{1}{4}}} = 0.707\,r$ Ans

b) $\tau = \frac{\rho}{c}\tau_{max} = \frac{\rho}{r}(\frac{2T}{\pi r^3}) = \frac{2T}{\pi r^4}\rho$; $\quad dA = 2\pi\rho\, d\rho$

$dT = \rho\tau\, dA = \rho[\frac{2T}{\pi r^4}\rho](2\pi\rho\, d\rho) = \frac{4T}{r^4}\rho^3 d\rho$

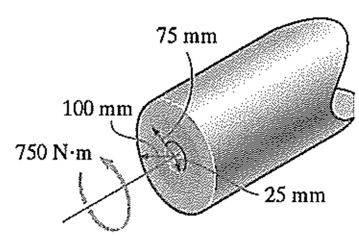


$\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int_0^{r'} \rho^3 d\rho$

$\frac{T}{4} = \frac{4T}{r^4}\frac{\rho^4}{4}\Big|_0^{r'}$; $\quad \frac{1}{4} = \frac{(r')^4}{r^4}$

$r' = 0.707\,r$ Ans

***5–4.** The tube is subjected to a torque of 750 N·m. Determine the amount of this torque that is resisted by the gray shaded section. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

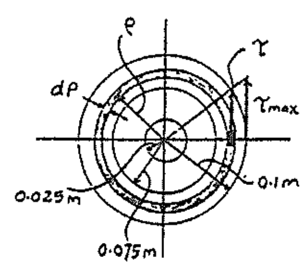


a) ***Applying Torsion Formula:***

$$\tau_{max} = \frac{Tc}{J} = \frac{750(0.1)}{\frac{\pi}{2}(0.1^4 - 0.025^4)} = 0.4793 \text{ MPa}$$

$$\tau_{max} = 0.4793(10^6) = \frac{T'(0.1)}{\frac{\pi}{2}(0.1^4 - 0.075^4)}$$

$$T' = 515 \text{ N}\cdot\text{m} \qquad \text{Ans}$$



b) ***Integration Method:***

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max} \quad \text{and} \quad dA = 2\pi\rho\, d\rho$$

$$dT' = \rho\tau\, dA = \rho\tau(2\pi\rho\, d\rho) = 2\pi\tau\rho^2\, d\rho$$

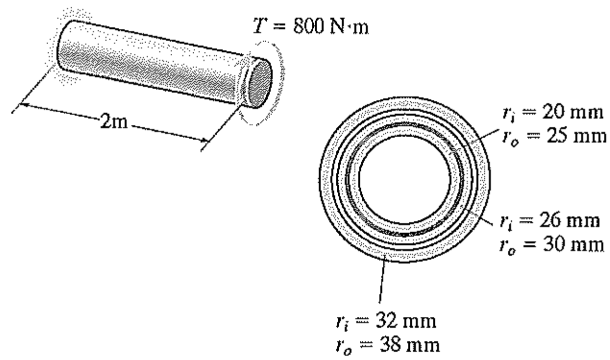
$$T' = \int 2\pi\tau\rho^2\, d\rho = 2\pi\int_{0.075\text{m}}^{0.1\text{m}} \tau_{max}\left(\frac{\rho}{c}\right)\rho^2\, d\rho$$

$$= \frac{2\pi\tau_{max}}{c}\int_{0.075\text{m}}^{0.1\text{m}} \rho^3\, d\rho$$

$$= \frac{2\pi(0.4793)(10^6)}{0.1}\left[\frac{\rho^4}{4}\right]\Bigg|_{0.075\text{m}}^{0.1\text{m}}$$

$$= 515 \text{ N}\cdot\text{m} \qquad \text{Ans}$$

5–11. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800\ \text{N}\cdot\text{m}$ is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.

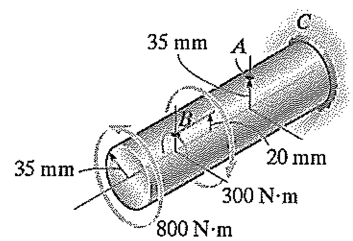


$$J = \frac{\pi}{2}((0.038)^4 - (0.032)^4) + \frac{\pi}{2}((0.030)^4 - (0.026)^4) + \frac{\pi}{2}((0.025)^4 - (0.020)^4)$$

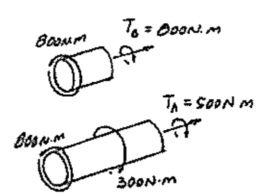
$$J = 2.545(10^{-6})\text{m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{800(0.038)}{2.545(10^{-6})} = 11.9\ \text{MPa} \qquad \textbf{Ans}$$

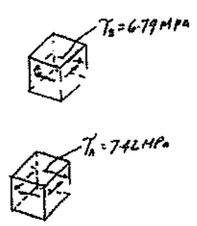
***5–12.** The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



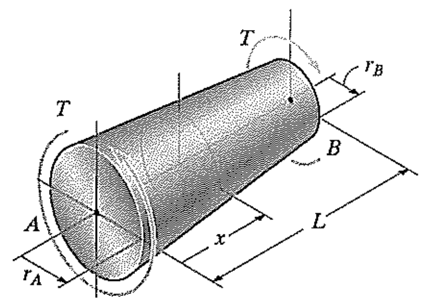
$$\tau_B = \frac{T_B \rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79\ \text{MPa} \qquad \textbf{Ans}$$



$$\tau_A = \frac{T_A\, c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42\ \text{MPa} \qquad \textbf{Ans}$$

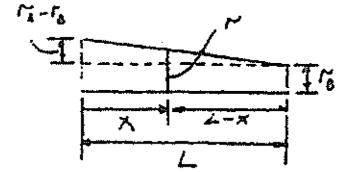


5–30. The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.



$$r = r_B + \frac{r_A - r_B}{L}(L - x) = \frac{r_B L + (r_A - r_B)(L - x)}{L}$$

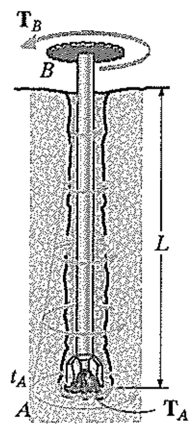
$$= \frac{r_A(L - x) + r_B x}{L}$$



$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

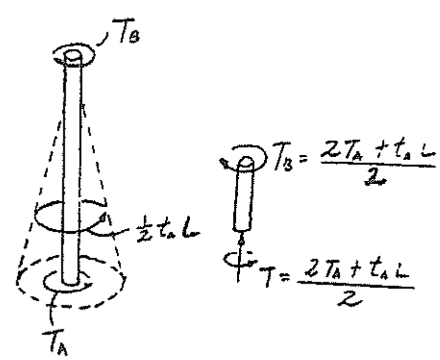
$$= \frac{2T}{\pi[\frac{r_A(L - x) + r_B x}{L}]^3} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3}$$ **Ans**

5–31. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



$$T_A + \frac{1}{2}t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2}$$ **Ans**

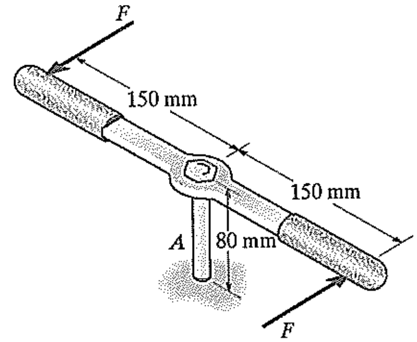


Maximum shear stress : The maximum torque is within the region above the distributed torque.

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau_{max} = \frac{[\frac{(2T_A + t_A L)}{2}](r_o)}{\frac{\pi}{2}(r_o^4 - r_i^4)} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)}$$ **Ans**

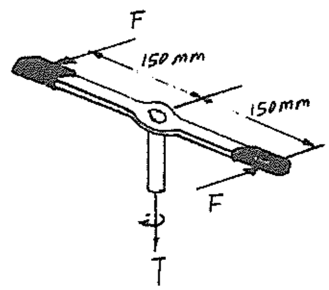
*5–52. The 8-mm-diameter A-36 bolt is screwed tightly into a block at A. Determine the couple forces F that should be applied to the wrench so that the maximum shear stress in the bolt becomes 18 MPa. Also, compute the corresponding displacement of each force F needed to cause this stress. Assume that the wrench is rigid.



$$T - F(0.3) = 0 \qquad (1)$$

$$\tau_{max} = \frac{Tc}{J}; \qquad 18(10^6) = \frac{T(0.004)}{\frac{\pi}{2}(0.004^4)}$$

$T = 1.8096\ \text{N}\cdot\text{m}$



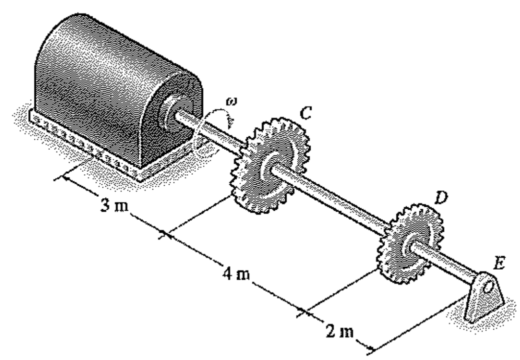
From Eq. (1),

$F = 6.03$ N **Ans**

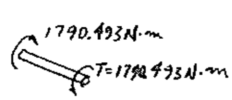
$$\phi = \frac{TL}{JG} = \frac{1.8096(0.08)}{\frac{\pi}{2}[(0.0040)^4]75(10^9)} = 0.00480 \text{ rad}$$

$s = r\phi = 0.15(0.00480) = 0.00072$ m $= 0.720$ mm **Ans.**

5–53. The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.

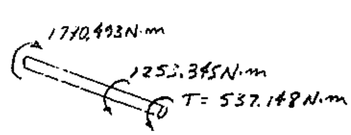


$$P = T\omega; \qquad 150(10^3)W = T(800\frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ sec}})(\frac{2\pi\text{ rad}}{1\text{ rev}})$$



$$T = 1790.493\ \text{N}\cdot\text{m}$$

$T_C = 1790.493(0.7) = 1253.345\ \text{N}\cdot\text{m}$
$T_D = 1790.493(0.3) = 537.148\ \text{N}\cdot\text{m}$



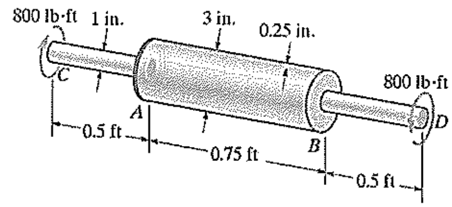
Maximum torque is in region BC.

$$\tau_{max} = \frac{Tc}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12 \text{ MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma(\frac{TL}{JG}) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

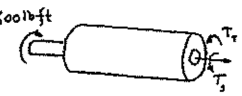
$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102 \text{ rad} = 0.585° \qquad \textbf{Ans}$$

5–78. The composite shaft consists of a mid-section that includes the 1-in.-diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 lb · ft. The material is A-36 steel.



Equilibrium :

$$800(12) - T_T - T_S = 0$$



Compatibility condition :

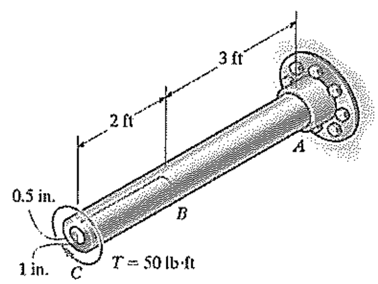
$$\phi_T = \phi_S; \qquad \frac{T_T(0.75)(12)}{\frac{\pi}{2}((1.5)^4 - (1.25)^4)G} = \frac{T_S(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$$T_T = 9376.42 \text{ lb}\cdot\text{in.}$$

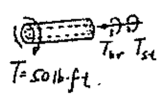
$$T_S = 223.58 \text{ lb}\cdot\text{in.}$$

$$\phi_{C/D} = \Sigma\frac{TL}{JG} = \frac{800(12)(1)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} + \frac{223.58(0.75)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.1085 \text{ rad} = 6.22° \quad \textbf{Ans}$$

5–79. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50$ lb · ft is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 11.5(10^3)$ ksi, $G_{br} = 5.6(10^3)$ ksi.



Equilibrium :



$$T_{br} + T_{st} - 50 = 0 \qquad (1)$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$

$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} = \frac{T_{st}(2)(12)}{\frac{\pi}{2}(1^4 - 0.5^4)(11.5)(10^6)}$$

$$T_{br} = 0.032464\, T_{st} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$T_{st} = 48.428$ lb · ft; $\quad T_{br} = 1.572$ lb · ft

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{1.572(12)(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} + \frac{50(12)(3)(12)}{\frac{\pi}{2}(1^4)(11.5)(10^6)}$$
$$= 0.002019 \text{ rad} = 0.116° \quad \textbf{Ans}$$

$$(\tau_{st})_{max\,AB} = \frac{T_{AB}c}{J} = \frac{50(12)(1)}{\frac{\pi}{2}(1^4)} = 382 \text{ psi}$$

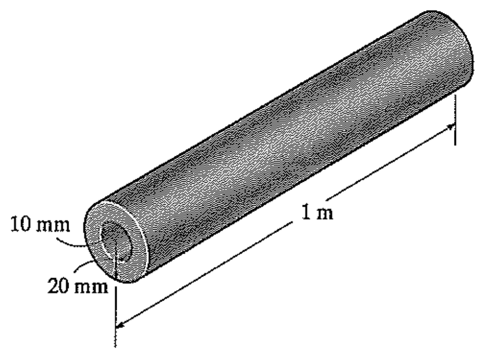
$$(\tau_{st})_{max\,BC} = \frac{T_{st}c}{J} = \frac{48.428(12)(1)}{\frac{\pi}{2}(1^4 - 0.5^4)} = 394.63 \text{ psi} = 395 \text{ psi (Max)} \quad \textbf{Ans}$$

$$(\gamma_{st})_{max} = \frac{(\tau_{st})_{max}}{G} = \frac{394.63}{11.5(10^6)} = 34.3(10^{-6}) \text{ rad} \quad \textbf{Ans}$$

$$(\tau_{br})_{max} = \frac{T_{br}c}{J} = \frac{1.572(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 96.07 \text{ psi} = 96.1 \text{ psi (Max)} \quad \textbf{Ans}$$

$$(\gamma_{br})_{max} = \frac{(\tau_{br})_{max}}{G} = \frac{96.07}{5.6(10^6)} = 17.2(10^{-6}) \text{ rad} \quad \textbf{Ans}$$

5–123. A tubular shaft has an inner diameter of 20 mm, an outer diameter of 40 mm, and a length of 1 m. It is made of an elastic perfectly plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum torque it can transmit. What is the angle of twist of one end with respect to the other end if the shear strain on the inner surface of the tube is about to yield? $G = 80$ GPa.



Plastic Torque :

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \, \rho^2 \, d\rho$$

$$= 2\pi \tau_Y \left[\frac{\rho^3}{3}\right]\Bigg|_{c_i}^{c_o}$$

$$= \frac{2\pi \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(100)(10^6)}{3}\left(0.02^3 - 0.01^3\right)$$

$$= 1466 \text{ N}\cdot\text{m} = 1.47 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of Twist :

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100(10^6)}{80(10^9)} = 0.00125 \text{ rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \left(\frac{0.00125}{0.01}\right)(1) = 0.125 \text{ rad} = 7.16° \qquad \textbf{Ans}$$