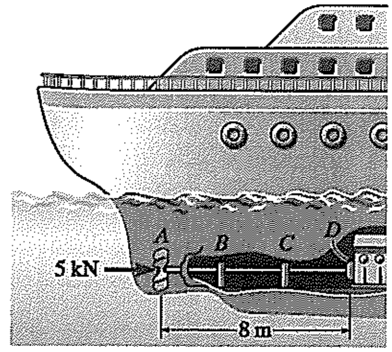
4–1. The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing D at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at B and C are journal bearings.

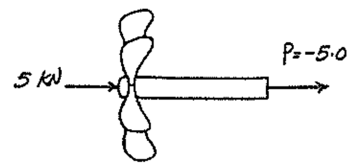


Internal Force : As shown on FBD.
Displacement :

$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}(0.4^2 - 0.3^2)\;200(10^9)}$$

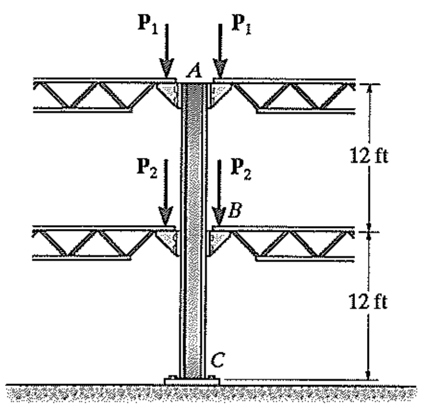
$$= -3.638(10^{-6})\ \text{m}$$

$$= -3.64(10^{-3})\ \text{mm} \qquad \textbf{Ans}$$



Negative sign indicates that end A moves towards end D.

4–2. The A-36 steel column is used to support the symmetric loads from the two floors of a building. Determine the vertical displacement of its top, A, if $P_1 = 40$ kip, $P_2 = 62$ kip, and the column has a cross-sectional area of 23.4 in^2.

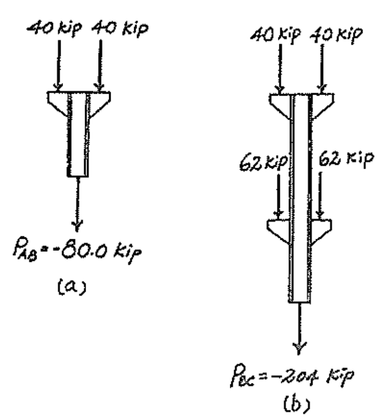


Internal Forces : As shown on FBD (a) and (b)
Displacement :

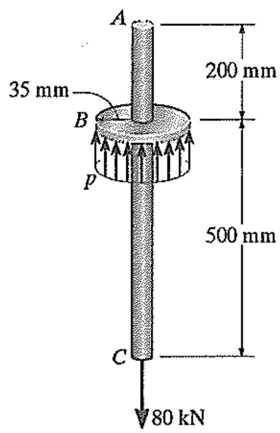
$$\delta_A = \sum \frac{PL}{AE} = \frac{-80.0\,(12)(12)}{23.4\,(29.0)(10^3)} + \frac{(-204)\,(12)(12)}{23.4\,(29.0)(10^3)}$$

$$= -0.0603\ \text{in.} \qquad \textbf{Ans}$$

Negative sign indicates that end A moves toward end C.



4–7. The 15-mm-diameter A-36 steel shaft AC is supported by a rigid collar, which is fixed to the shaft at B. If it is subjected to an axial load of 80 kN at its end, determine the uniform pressure distribution p on the collar required for equilibrium. Also, what is the elongation on segment BC and segment BA?



Equations of Equilibrium : FBD (a)

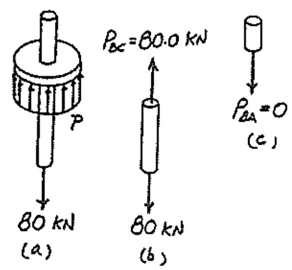
$$+\uparrow \Sigma F_y = 0; \quad p\left[\frac{\pi}{4}\left(0.07^2 - 0.015^2\right)\right] - 80(10^3) = 0$$

$$p = 21.79(10^6)\ \text{Pa} = 21.8\ \text{MPa} \qquad \textbf{Ans}$$

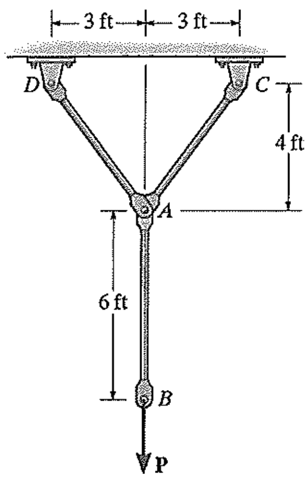
Internal Forces : As shown on FBD (b) and (c).
Displacement :

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{80.0(10^3)(500)}{\frac{\pi}{4}(0.015^2)(200)(10^9)} = 1.13\ \text{mm} \qquad \textbf{Ans}$$

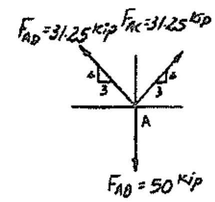
$$\delta_{BA} = \frac{P_{BA}L_{BA}}{A_{BA}E} = 0 \qquad \textbf{Ans}$$



***4–16.** The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of $0.730\ \text{in}^2$. If a vertical force of $P = 50$ kip is applied to the end B of member AB, determine the vertical displacement of point B.



$$\delta_{A/D} = \delta_{A/C} = \frac{PL}{AE} = \frac{31.25(5)(12)}{(0.730)(29)(10^3)} = 0.08857 \text{ in.}$$



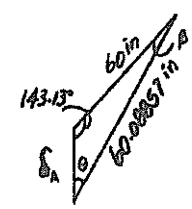
$$\delta_{B/A} = \frac{PL}{AE} = \frac{50(6)(12)}{(0.730)(29)(10^3)} = 0.17005 \text{ in.}$$

$$\phi = 90° + \tan^{-1}\left(\frac{4}{3}\right) = 143.13°$$

$$\frac{\sin\theta}{60} = \frac{\sin 143.13°}{60.08857}; \quad \theta = 36.806584°$$

$$\beta = 180° - 36.806584° - 143.130102° = 0.06331297°$$

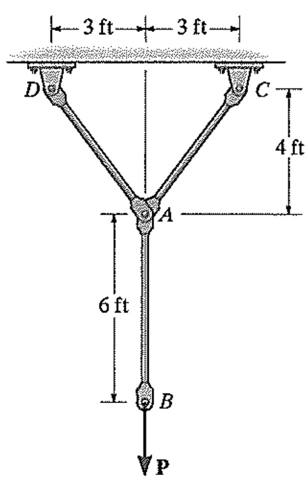
$$\frac{\delta_A}{\sin 0.06331297°} = \frac{60}{\sin 36.806584°}$$

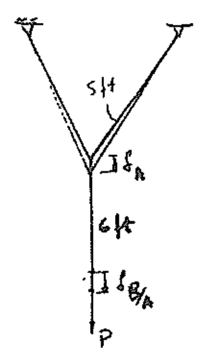


$$\delta_A = 0.11066 \text{ in.}$$

$$\delta_B = \delta_A + \delta_{B/A} = 0.11066 + 0.17005 = 0.281 \text{ in.} \qquad \textbf{Ans}$$

4–17. The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of $0.75\ \text{in}^2$. Determine the magnitude of the force **P** needed to displace point B 0.10 in. downward.





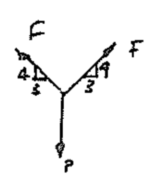
$$\delta_B = \delta_A + \delta_{B/A} = 0.10 \text{ in.} \qquad (1)$$

$$\delta_{B/A} = \frac{PL}{AE} = \frac{P(6)(12)}{(0.75)(29)(10^3)} = 0.0033103P$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F\left(\frac{4}{5}\right) - P = 0$$

$$F = 0.625\,P$$

$$\delta_{A/C} = \delta_{A/D} = \frac{0.625P(5)(12)}{(0.75)(29)(10^3)} = 0.0017241P$$



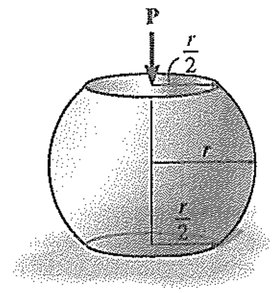
$$\delta_A = \delta_{A/C}\left(\frac{5}{4}\right) = 0.0021552P$$

From Eq. (1),

$$0.0033103P + 0.0021552P = 0.10$$

$$P = 18.3 \text{ kip} \qquad \textbf{Ans.}$$

4–27. The ball is truncated at its ends and is used to support the bearing load **P**. If the modulus of elasticity for the material is E, determine the decrease in its height when the load is applied.

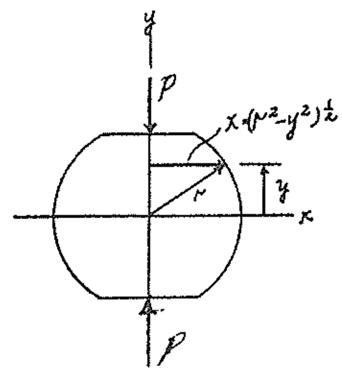


Displacement :

Geometry :

$$A\,(y) = \pi x^2 = \pi\,(r^2 - y^2)$$

Displacement : When $x = \dfrac{r}{2}$, $\quad y = \pm\dfrac{\sqrt{3}}{2}r$



$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

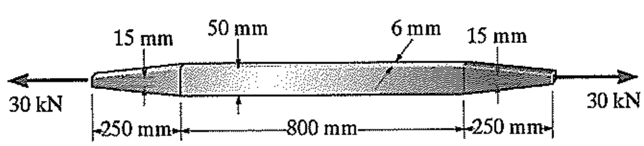
$$= \frac{P}{\pi E}\int_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r} \frac{dy}{r^2 - y^2}$$

$$= \frac{P}{\pi E}\left[\frac{1}{2r}\ln\frac{r+y}{r-y}\right]\Bigg|_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r}$$

$$= \frac{P}{2\pi rE}\,[\,\ln 13.9282 - \ln 0.07180\,]$$

$$= \frac{2.63\,P}{\pi rE} \qquad \textbf{Ans}$$

*4–28. Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa.

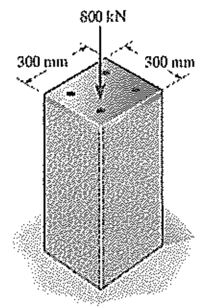


$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}\left(\ln\frac{50}{15}\right) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$= 2.37$ mm **Ans**

4–35. The column is constructed from high-strength concrete and four A-36 steel reinforcing rods. If it is subjected to an axial force of 800 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the steel and three-fourths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Equilibrium : Require $P_{st} = \frac{1}{4}(800) = 200$ kN and

$P_{con} = \frac{3}{4}(800) = 600$ kN.

Compatibility :

$$\delta_{con} = \delta_{st}$$

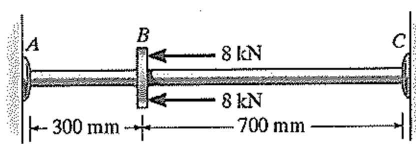
$$\frac{P_{con}L}{(0..3^2 - A_{st})(25.0)(10^9)} = \frac{P_{st}L}{A_{st}(200)(10^9)}$$

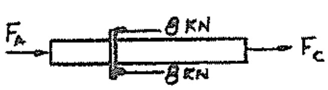
$$A_{st} = \frac{0.09P_{st}}{8P_{con} + P_{st}}$$

$$4\left[\left(\frac{\pi}{4}\right)d^2\right] = \frac{0.09(200)}{8(600) + 200}$$

$$d = 0.03385\text{m} = 33.9\text{mm}$$ **Ans**

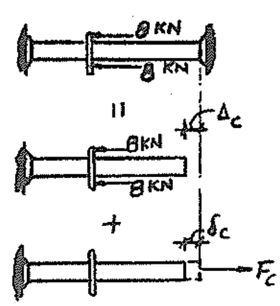
***4–36.** The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.





$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0 \qquad$ **(1)**

By superposition :



$(\overset{+}{\rightarrow}) \qquad 0 = -\Delta_C + \delta_C$

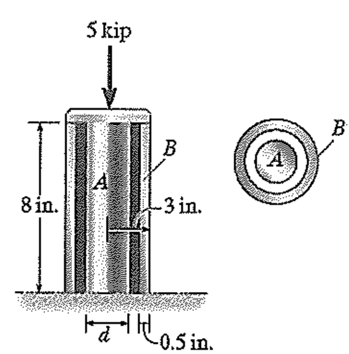
$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$

$F_C = 4.80$ kN **Ans**

From Eq. (1),

$F_A = 11.2$ kN **Ans**

4–37. The 304 stainless steel post A has a diameter of $d = 2$ in. and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



Equations of Equilibrium :

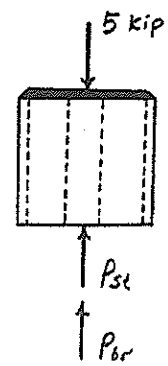
$$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{br} - 5 = 0 \qquad [1]$$

Compatibility :

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(8)}{\frac{\pi}{4}(2^2)(28.0)(10^3)} = \frac{P_{br}(8)}{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}$$

$$P_{st} = 0.69738\, P_{br} \qquad [2]$$



Solving Eqs. [1] and [2] yields :

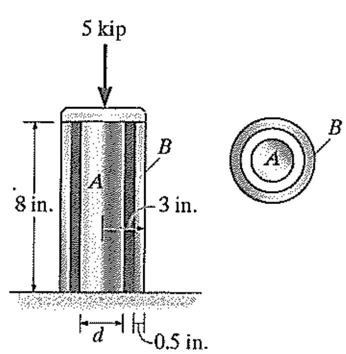
$$P_{br} = 2.9457 \text{ kip} \qquad P_{st} = 2.0543 \text{ kip}$$

Average Normal Stress :

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{2.9457}{\frac{\pi}{4}(6^2 - 5^2)} = 0.341 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{2.0543}{\frac{\pi}{4}(2^2)} = 0.654 \text{ ksi} \qquad \textbf{Ans}$$

4–38. The 304 stainless steel post A is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the required diameter d of the steel post so that the load is shared equally between the post and tube.



Equilibrium : The force of 60 kip is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 2.50 \text{ kip}$$

Compatibility :

$$\delta_{st} = \delta_{br}$$

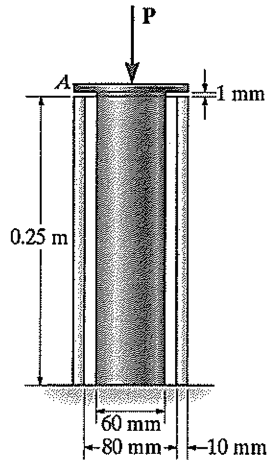
$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

$$\left(\frac{\pi}{4}\right)d^2 = \frac{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}{28.0(10^3)}$$

$$d = 2.39 \text{ in.} \qquad \textbf{Ans}$$

4–41. The support consists of a solid red brass C83400 post surrounded by a 304 stainless steel tube. Before the load is applied, the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.

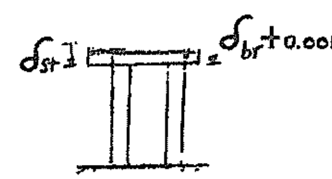


Require,

$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi[(0.05)^2 - (0.04)^2]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$

$$0.45813\, F_{st} = 0.87544\, F_{br} + 10^6 \qquad (1)$$



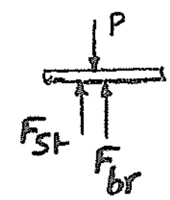
$$+\uparrow \Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad (2)$$

Assume brass yields, then

$$(F_{br})_{max} = \sigma_Y A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3 \text{ N}$$

$$(\varepsilon_Y)_{br} = \sigma_Y / E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3}) \text{ mm/mm}$$

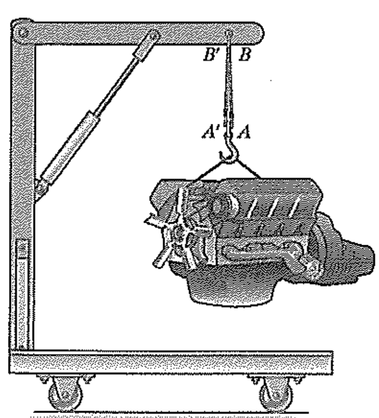
$$\delta_{br} = (\varepsilon_Y)_{br} L = 0.6931(10^{-3})(0.25) = 0.1733 \text{ mm} < 1 \text{ mm}$$



Thus only the brass is loaded.

$$P = F_{br} = 198 \text{ kN} \qquad \text{Ans}$$

4–42. Two A-36 steel wires are used to support the 650-lb engine. Originally, AB is 32 in. long and $A'B'$ is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in^2.

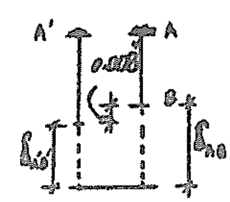


$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad (1)$$

$$\delta_{AB} = \delta_{A'B'} + 0.008$$

$$\frac{T_{AB}(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}(32.008)}{(0.01)(29)(10^6)} + 0.008$$

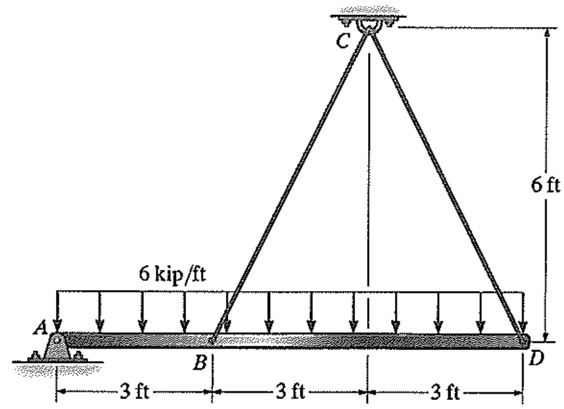
$$32T_{AB} - 32.008T_{A'B'} = 2320$$



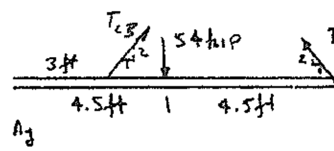
$$T_{AB} = 361 \text{ lb} \qquad \text{Ans}$$

$$T_{A'B'} = 289 \text{ lb} \qquad \text{Ans}$$

***4–68.** The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in^2, and $E = 31(10^3)$ ksi.



$$\circlearrowleft + \Sigma M_A = 0; \quad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(3) - 54(4.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)9 = 0 \qquad (1)$$



$$\theta = \tan^{-1}\frac{6}{6} = 45°$$

$$L^2_{B'C} = (3)^2 + (8.4853)^2 - 2(3)(8.4853)\cos\theta'$$

Also,

$$L^2_{D'C} = (9)^2 + (8.4853)^2 - 2(9)(8.4853)\cos\theta' \qquad (2)$$

Thus, eliminating $\cos\theta'$,

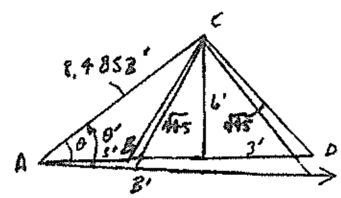
$$-L^2_{B'C}(0.019642) + 1.5910 = -L^2_{D'C}(0.0065473) + 1.001735$$

$$L^2_{B'C}(0.019642) = 0.0065473\,L^2_{D'C} + 0.589256$$

$$L^2_{B'C} = 0.333\,L^2_{D'C} + 30$$

But,

$$L_{B'C} = \sqrt{45} + \delta_{BC}, \qquad L_{D'C} = \sqrt{45} + \delta_{DC}$$



Neglect squares or δ's since small strain occurs.

$$L^2_{B'C} = (\sqrt{45} + \delta_{BC})^2 = 45 + 2\sqrt{45}\,\delta_{BC}$$

$$L^2_{D'C} = (\sqrt{45} + \delta_{DC})^2 = 45 + 2\sqrt{45}\,\delta_{DC}$$

$$45 + 2\sqrt{45}\,\delta_{BC} = 0.333(45 + 2\sqrt{45}\,\delta_{DC}) + 30$$

$$2\sqrt{45}\,\delta_{BC} = 0.333(2\sqrt{45})\,\delta_{DC}$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{45}}{AE} = 3\frac{T_{CB}\sqrt{45}}{AE}$$

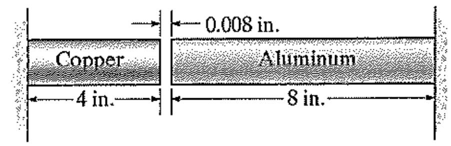
$$T_{CD} = 3\,T_{CB}$$

From Eq. (1),

$T_{CD} = 27.1682$ kip $= 27.2$ kip **Ans**

$T_{CB} = 9.06$ kip **Ans**

4–78. The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when $T_1 = 60°F$. Each rod has a diameter of 1.25 in., $\alpha_{al} = 13(10^{-6})/°F$, $E_{al} = 10(10^3)$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°F$, $E_{cu} = 18(10^3)$ ksi. Determine the average normal stress in each rod if $T_2 = 300°F$, and also calculate the new length of the aluminum segment.

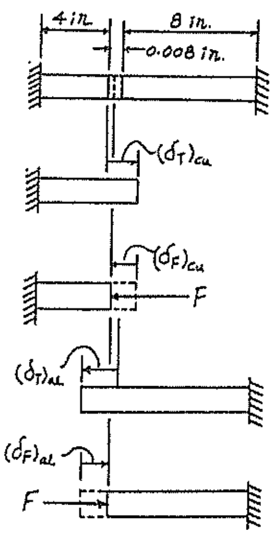


Compatibility :

$$0.008 = (\delta_T)_{cu} - (\delta_F)_{cu} + (\delta_T)_{al} - (\delta_F)_{al}$$

$$0.008 = 9.4(10^{-6})(300-60)(4) - \frac{F(4)}{\frac{\pi}{4}(1.25^2)(18)(10^3)} + 13(10^{-6})(300-60)(8) - \frac{F(8)}{\frac{\pi}{4}(1.25^2)(10)(10^3)}$$

$$F = 31.194 \text{ kip}$$



Average Normal Stress :

$$\sigma_{al} = \sigma_{cu} = \frac{F}{A} = \frac{31.194}{\frac{\pi}{4}(1.25^2)} = 25.4 \text{ ksi} \qquad \textbf{Ans}$$

Displacement :

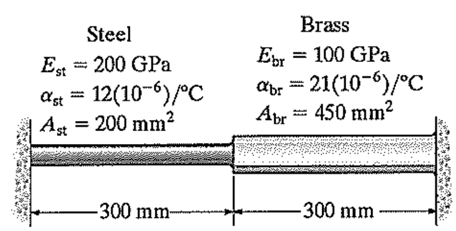
$$\delta_{al} = (\delta_T)_{al} - (\delta_F)_{al}$$

$$= 13(10^{-6})[300-60](8) - \frac{31.194(8)}{\frac{\pi}{4}(1.25^2)(10)(10^3)}$$

$$= 0.0046247 \text{ in.}$$

$$L'_{al} = L_{al} + \delta_{al} = 8 + 0.0046247 = 8.00462 \text{ in.} \qquad \textbf{Ans}$$

4–79. Two bars, each made of a different material, are connected and placed between two walls when the temperature is $T_1 = 10°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 20°C$. The material properties and cross-sectional area of each bar are given in the figure.

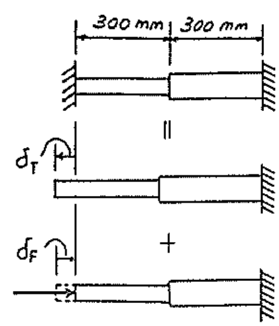


Compatibility :

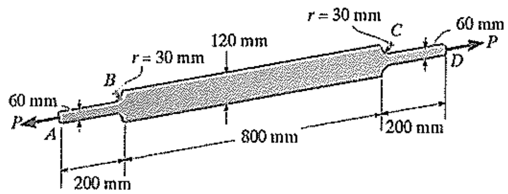
$$(\overset{+}{\leftarrow}) \qquad 0 = \delta_T - \delta_F$$

$$0 = 12(10^{-6})(20-10)(0.3) + 21(10^{-6})(20-10)(0.3) - \frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.3)}{450(10^{-6})(100)(10^9)}$$

$$F = 6988.2 \text{ N} = 6.99 \text{ kN} \qquad \textbf{Ans}$$



4–95. The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at B and C, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Compute its elongation neglecting the effect of the fillets.



Maximum Normal Stress at fillet :

$$\frac{r}{h} = \frac{30}{60} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86 \text{ N} = 77.1 \text{ kN} \qquad \textbf{Ans}$$

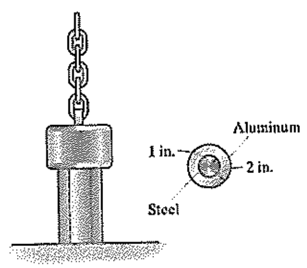
Displacement :

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429 \text{ mm} \qquad \textbf{Ans}$$

***4–96.** The 300-kip weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.



Equations of Equilibrium :

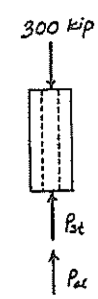
$$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{al} - 300 = 0 \qquad [1]$$

Elastic Analysis: Assume both materials still behave elastically under the load.

$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(2)^2(29)(10^3)} = \frac{P_{al}L}{\frac{\pi}{4}(4^2 - 2^2)(10.6)(10^3)}$$

$$P_{st} = 0.9119\, P_{al}$$



Solving Eqs. [1] and [2] yields :

$$P_{al} = 156.91 \text{ kip} \qquad P_{st} = 143.09 \text{ kip}$$

Average Normal Stress :

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{156.91}{\frac{\pi}{4}(4^2 - 2^2)}$$

$$= 16.65 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi} \;\; (OK!)$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{143.09}{\frac{\pi}{4}(2^2)}$$

$$= 45.55 \text{ ksi} > (\sigma_Y)_{st} = 36.0 \text{ ksi}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$$\sigma_{st} = (\sigma_Y)_{st} = 36.0 \text{ ksi} \qquad \textbf{Ans}$$

$$P_{st} = (\sigma_Y)_{st}A_{st} = 36.0\left(\frac{\pi}{4}\right)(2^2) = 113.10 \text{ kip}$$

From Eq. [1] $\quad P_{al} = 186.90$ kip

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{186.90}{\frac{\pi}{4}(4^2 - 2^2)} = 19.83 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi}$$

Then $\qquad \sigma_{al} = 19.3$ ksi $\qquad$ **Ans**