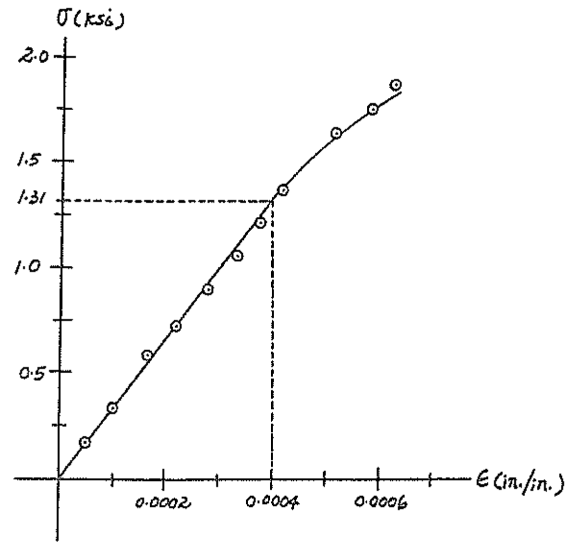
3–1. A concrete cylinder having a diameter of 6.00 in. and gauge length of 12 in. is tested in compression. The results of the test are reported in the table as load versus contraction. Draw the stress–strain diagram using scales of 1 in. = 0.5 ksi and 1 in. = $0.2(10^{-3})$ in./in. From the diagram, determine approximately the modulus of elasticity.

Load (kip)	Contraction (in.)
0	0
5.0	0.0006
9.5	0.0012
16.5	0.0020
20.5	0.0026
25.5	0.0034
30.0	0.0040
34.5	0.0045
38.5	0.0050
46.5	0.0062
50.0	0.0070
53.0	0.0075

Stress and Strain :

$\sigma = \frac{P}{A}$ (ksi)	$\varepsilon = \frac{\delta L}{L}$ (in./in.)
0	0
0.177	0.00005
0.336	0.00010
0.584	0.000167
0.725	0.000217
0.902	0.000283
1.061	0.000333
1.220	0.000375
1.362	0.000417
1.645	0.000517
1.768	0.000583
1.874	0.000625

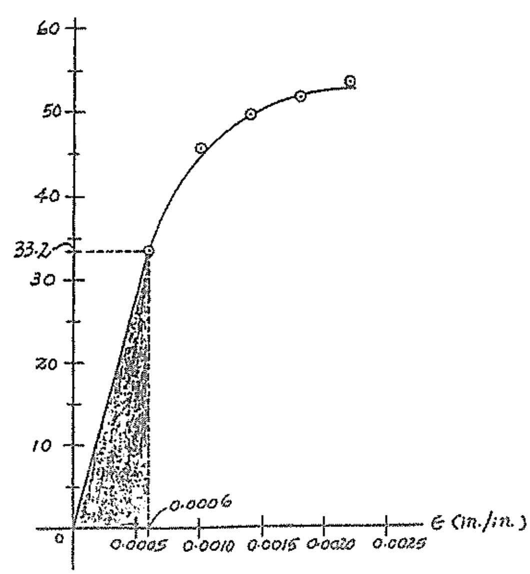


Modulus of Elasticity : From the stress – strain diagram

$$E_{approx} = \frac{1.31 - 0}{0.0004 - 0} = 3.275(10^3)\text{ ksi} \qquad \text{Ans}$$

3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022



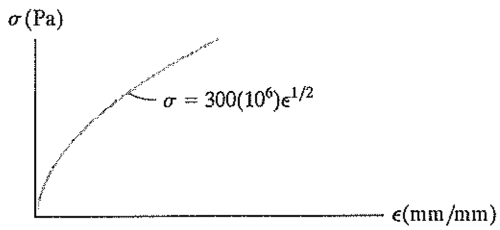
Modulus of Elasticity : From the stress – strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3(10^3)\text{ ksi} \qquad \text{Ans}$$

Modulus of Resilience : The modulus of resilience is equal to the area under the *linear portion* of the stress – strain diagram (shown shaded).

$$u_r = \frac{1}{2}(33.2)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\,\frac{\text{in.}}{\text{in.}}\right) = 9.96\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \text{Ans}$$

*3–12. Fiberglass has a stress–strain diagram as shown. If a 50-mm-diameter bar of length 2 m made from this material is subjected to an axial tensile load of 60 kN, determine its elongation.



$$\sigma = \frac{P}{A} = \frac{60(10^3)}{\pi(0.025)^2} = 30.558 \text{ MPa}$$

$$\sigma = 300(10^6)\varepsilon^{1/2}$$

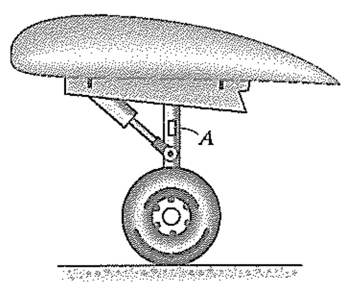
$$30.558(10^6) = 300(10^6)\varepsilon^{1/2}$$

$$\varepsilon = 0.010375 \text{ mm/mm}$$

$$\delta = L\varepsilon = 2(0.010375) = 0.0208 \text{ m}$$

$\delta = 20.8$ mm **Ans**

3–13. The change in weight of an airplane is determined from reading the strain gauge A mounted in the plane's aluminum wheel strut. *Before* the plane is loaded, the strain-gauge reading in a strut is $\epsilon_1 = 0.00100$ in./in., whereas after loading $\epsilon_2 = 0.00243$ in./in. Determine the change in the force on the strut if the cross-sectional area of the strut is 3.5 in². $E_{al} = 10(10^3)$ ksi.



Stress - Strain Relationship : Applying Hooke's law $\sigma = E\varepsilon$.

$$\sigma_1 = 10(10^3)(0.00100) = 10.0 \text{ ksi}$$
$$\sigma_2 = 10(10^3)(0.00243) = 24.3 \text{ ksi}$$

Normal Force : Applying equation $\sigma = \frac{P}{A}$.

$$P_1 = 10.0(3.5) = 35.0 \text{ kip}$$
$$P_2 = 24.3(3.5) = 85.05 \text{ kip}$$

$\Delta P = P_2 - P_1 = 85.05 - 35.0 = 50.0$ kip **Ans**

3–14. A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased to 1800 lb, the specimen elongates 0.9 in. Determine the modulus of elasticity for the material if it remains elastic.

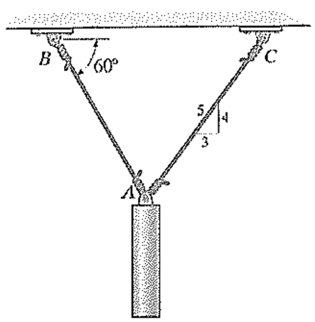
$$\sigma_1 = \frac{P}{A} = \frac{500}{\frac{\pi}{4}(0.5)^2} = 2.546 \text{ ksi}$$

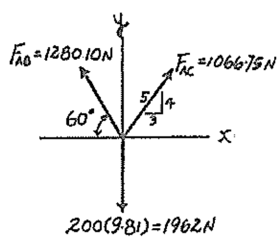
$$\sigma_2 = \frac{P}{A} = \frac{1800}{\frac{\pi}{4}(0.5)^2} = 9.167 \text{ ksi}$$

$$\Delta\varepsilon = \frac{0.9}{12} = 0.075 \text{ in./in.}$$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{9.167 - 2.546}{0.075} = 88.3 \text{ ksi}$$

3–18. The steel wires AB and AC support the 200-kg mass. If the allowable axial stress for the wires is $\sigma_{allow} = 130$ MPa, determine the required diameter of each wire. Also, what is the new length of wire AB after the load is applied? Take the unstretched length of AB to be 750 mm. $E_{st} = 200$ GPa.





Axial Force : The axial forces exerted by wires AB and AC are shown on FBD.

Allowable Normal Stress :

For wire AB

$$\sigma_{allow} = 130\left(10^6\right) = \frac{1280.10}{\frac{\pi}{4}(d_{AB}^2)}$$

$$d_{AB} = 0.003541 \text{ m} = 3.54 \text{ mm} \qquad \textbf{Ans}$$

For wire AC

$$\sigma_{allow} = 130\left(10^6\right) = \frac{1066.75}{\frac{\pi}{4}(d_{AC}^2)}$$

$$d_{AC} = 0.003232 \text{ m} = 3.23 \text{ mm} \qquad \textbf{Ans}$$

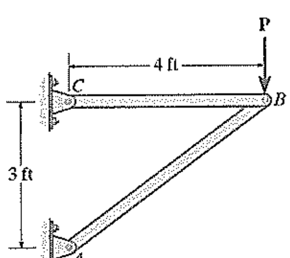
Stress – Strain Relationship : Applying Hooke's law

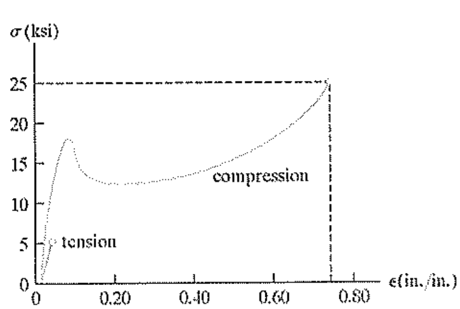
$$\varepsilon_{AB} = \frac{\sigma}{E} = \frac{130(10^6)}{200(10^9)} = 0.000650 \text{ mm/mm}$$

Thus,

$$L_{AB} = (L_{AB})_0 + \varepsilon_{AB}(L_{AB})_0$$
$$= 750 + 750(0.000650) = 750.49 \text{ mm} \qquad \textbf{Ans}$$

3–19. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 1.5 in^2 and BC is 4 in^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.



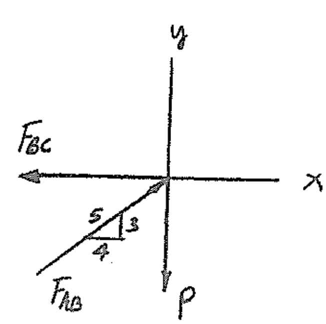


$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AB} - P = 0; \quad F_{AB} = 1.6667\,P \qquad [1]$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{BC} - \frac{4}{5}(1.6667P) = 0; \quad F_{BC} = 1.333\,P \qquad [2]$$

Assuming failure of bar BC :
From the stress - strain diagram $(\sigma_R)_t = 5$ ksi

$$\sigma = \frac{F_{BC}}{A_{BC}}; \quad 5 = \frac{F_{BC}}{4}; \quad F_{BC} = 20.0 \text{ kip}$$

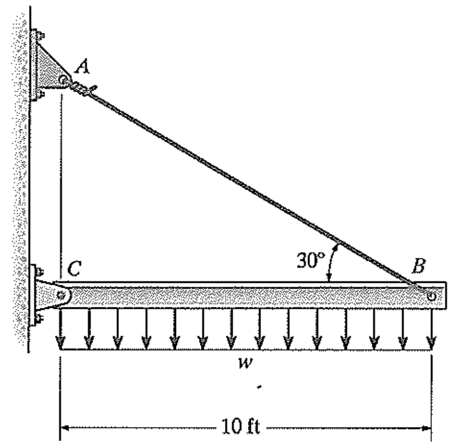


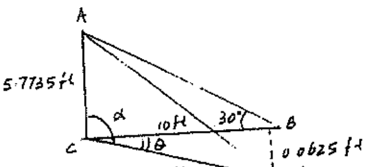
From Eq.[2], $P = 15.0$ kip
Assuming failure of bar AB :
From stress - strain diagram $(\sigma_R)_c = 25.0$ ksi

$$\sigma = \frac{F_{AB}}{A_{AB}}; \quad 25.0 = \frac{F_{AB}}{1.5}; \quad F_{AB} = 37.5 \text{ kip}$$

From Eq.[1], $P = 22.5$ kip
Choose the smallest value
$P = 15.0$ kip **Ans**

*3–24. The beam is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine the distributed load w if the end B is displaced 0.75 in. downward.





$$\sin\theta = \frac{0.0625}{10}; \qquad \theta = 0.3581°$$

$$\alpha = 90 + 0.3581° = 90.3581°$$

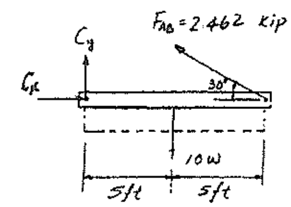
$$AB = \frac{10}{\cos 30°} = 11.5470 \text{ ft}$$

$$AB' = \sqrt{10^2 + 5.7735^2 - 2(10)(5.7735)\cos 90.3581°} = 11.5782 \text{ ft}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{11.5782 - 11.5470}{11.5470} = 0.002703 \text{ in./in.}$$

$$\sigma_{AB} = E\,\varepsilon_{AB} = 29(10^3)(0.002703) = 78.38 \text{ ksi}$$

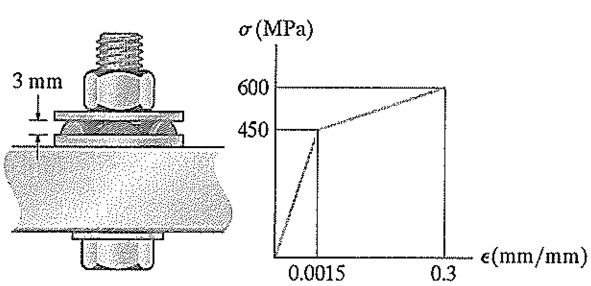
$$F_{AB} = \sigma_{AB} A_{AB} = 78.38\left(\frac{\pi}{4}\right)(0.2)^2 = 2.462 \text{ kip}$$



$$+\Sigma M_C = 0; \qquad 2.462 \sin 30°(10) - 10w(5) = 0;$$

$$w = 0.246 \text{ kip/ft} \qquad \textbf{Ans}$$

3–25. Direct tension indicators are sometimes used instead of torque wrenches to insure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six heads of the indicator that were originally 3 mm high are crushed 0.3 mm, leaving a contact area on each head of 1.5 mm², determine the tension in the bolt shank. The material has the stress–strain diagram shown.



Stress – Strain Relatioship : From the stress – strain diagram with $\varepsilon = \frac{0.3}{3} = 0.1 \text{ mm/mm} > 0.0015 \text{ mm/mm}$

$$\frac{\sigma - 450}{0.1 - 0.0015} = \frac{600 - 450}{0.3 - 0.0015}$$

$$\sigma = 499.497 \text{ MPa}$$

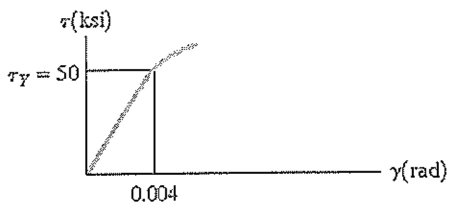
Axial Force : For each head

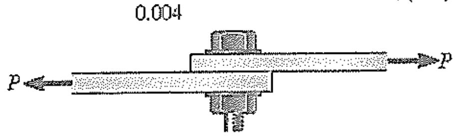
$$P = \sigma A = 499.4971\left(10^6\right)(1.5)\left(10^{-6}\right) = 749.24 \text{ N}$$

Thus, the tension in the bolt is

$$T = 6P = 6(749.24) = 4495 \text{ N} = 4.50 \text{ kN} \qquad \text{Ans}$$

3–31. The shear stress–strain diagram for a steel alloy is shown in the figure. If a bolt having a diameter of 0.25 in. is made of this material and used in the lap joint, determine the modulus of elasticity E and the force P required to cause the material to yield. Take $\nu = 0.3$.





Modulus of Rigidity : From the stress – strain diagram,

$$G = \frac{50}{0.004} = 12.5(10^3)\text{ ksi}$$

Modulus of Elasticity :

$$G = \frac{E}{2(1+\nu)}$$

$$12.5(10^3) = \frac{E}{2(1+0.3)}$$

$$E = 32.5(10^3)\text{ ksi} \qquad \textbf{Ans}$$

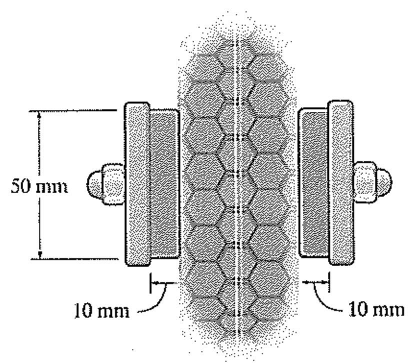
Yielding Shear : The bolt is subjected to a yielding shear of $V_Y = P$. From the stress – strain diagram, $\tau_Y = 50$ ksi

$$\tau_Y = \frac{V_Y}{A}$$

$$50 = \frac{P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 2.45\text{ kip} \qquad \textbf{Ans}$$

***3–32.** The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



Average Shear Stress : The shear force is $V = 50$ N.

$$\tau = \frac{V}{A} = \frac{50}{0.02(0.05)} = 50.0\text{ kPa}$$

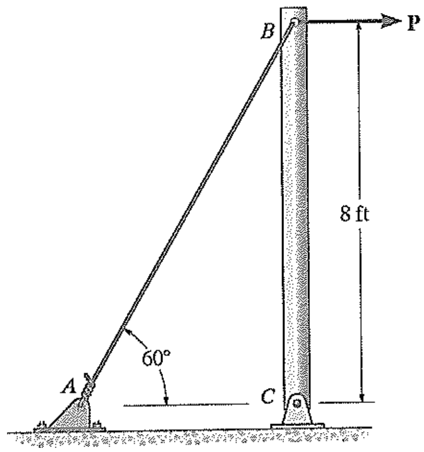
Shear Stress – Strain Relationship : Applying Hooke's law for shear

$$\tau = G\gamma$$

$$50.0(10^3) = 0.2(10^6)\ \gamma$$

$$\gamma = 0.250\text{ rad} \qquad \textbf{Ans}$$

3–39. The rigid pipe is supported by a pin at C and an A-36 guy wire AB. If the wire has a diameter of 0.2 in., determine the load P if the end B is displaced 0.10 in. to the right. $E_{st} = 29(10^3)$ ksi.



Geometry :

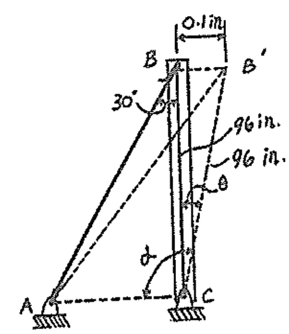
$$\sin\theta = \frac{0.1}{96} \qquad \theta = 0.05968°$$

$$\alpha = 90° + 0.05968° = 90.05968°$$

$$AC = 96\tan 30° = 55.4256 \text{ in}$$

$$AB = \frac{96}{\cos 30°} = 110.8513 \text{ in}$$

$$AB' = \sqrt{96^2 + 55.4256^2 - 2(96)(55.4256)\cos 90.05968°}$$
$$= 110.9012 \text{ in.}$$



Normal Stress and Strain :

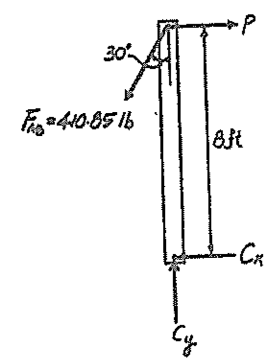
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$
$$= \frac{110.9012 - 110.8513}{110.8513}$$
$$= 0.4510(10^{-3}) \text{ in./in.}$$

Applying Hooke's law

$$\sigma_{AB} = E\varepsilon_{AB} = 29.0(10^3)\,0.4510(10^{-3}) = 13.08 \text{ ksi}$$

Thus,

$$F_{AB} = \sigma_{AB}A = 13.08(10^3)\left[\frac{\pi}{4}(0.2^2)\right] = 410.85 \text{ lb}$$



Equation of Equilibrium :

$$\curvearrowleft + \Sigma M_C = 0; \qquad 410.85\sin 30°(8) - P(8) = 0$$
$$P = 205 \text{ lb} \qquad \textbf{Ans}$$

***3–40.** While undergoing a tension test, a copper-alloy specimen having a gauge length of 2 in. is subjected to a strain of 0.40 in./in. when the stress is 70 ksi. If $\sigma_Y = 45$ ksi when $\epsilon_Y = 0.0025$ in./in., determine the distance between the gauge points when the load is released.

Elastic recovery $= 70\frac{(0.0025)}{45} = 0.0038889$ in./in.

Permanent set $= 0.4 - 0.0038889 = 0.3961$ in./in.

$\delta = 0.3961(2) = 0.792$ in.

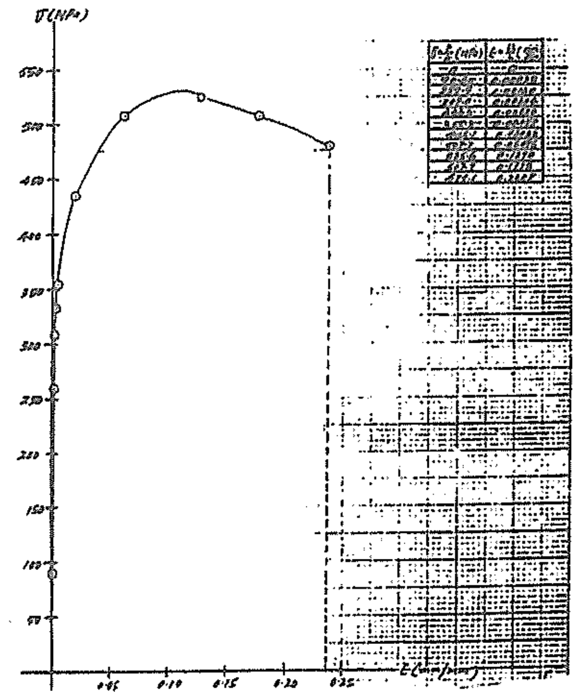
$L = 2 + 0.792 = 2.792$ in. **Ans**

3–43. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. Using the data listed in the table, plot the stress–strain diagram and determine approximately the modulus of toughness. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm.

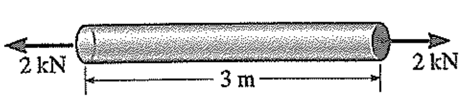
Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

The modulus of toughness = Total area under the curve. By counting squares we have (approximately)

$$u_t = (188.5 \text{ squares})\left(25\text{x}10^6 \frac{\text{N}}{\text{m}^2}\right)\left(0.025 \frac{\text{m}}{\text{m}}\right) = 118\left(10^6\right)\frac{\text{N}}{\text{m}^2}$$ **Ans**



***3–44.** An 8-mm-diameter brass rod has a modulus of elasticity of E_{br} = 100 GPa. If it is 3 m long and subjected to an axial load of 2 kN, determine its elongation. What is its elongation under the same load if its diameter is 6 mm?



$$\sigma = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.008^2)} = 39.789 \text{ MPa}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{39.789(10^6)}{100(10^9)} = 0.00039789$$

$$\delta = \varepsilon L = 0.00039789(3000) = 1.19 \text{ mm}$$ **Ans**

$$\sigma' = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.006^2)} = 70.735 \text{ MPa}$$

$$\varepsilon' = \frac{\sigma}{E} = \frac{70.735(10^6)}{100(10^9)} = 0.00070735$$

$$\delta' = \varepsilon' L = 0.00070735(3000) = 2.12 \text{ mm}$$ **Ans**