2–1. An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6$ in.

$d = 7$ in.

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7-6}{6} = 0.167 \text{ in./in.}$$ **Ans**

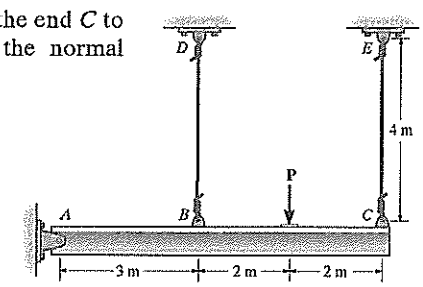
2–2. A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

$L_0 = 15$ in.

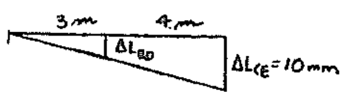
$L = \pi(5 \text{ in.})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$ **Ans**

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

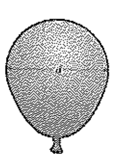


$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans**

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans**

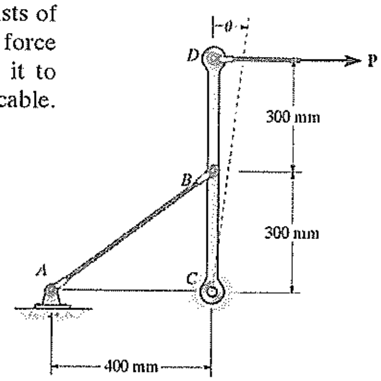
*2–4. The center portion of the rubber balloon has a diameter of $d = 4$ in. If the air pressure within it causes the balloon's diameter to become $d = 5$ in., determine the average normal strain in the rubber.



Average Normal Strain :

$$\varepsilon_{avg} = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{d - d_0}{d_0} = \frac{5-4}{4} = 0.250 \text{ in./in.}$$ **Ans**

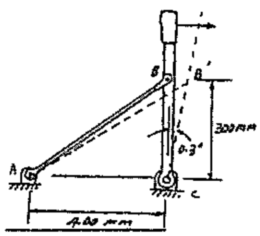
***2–8.** Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

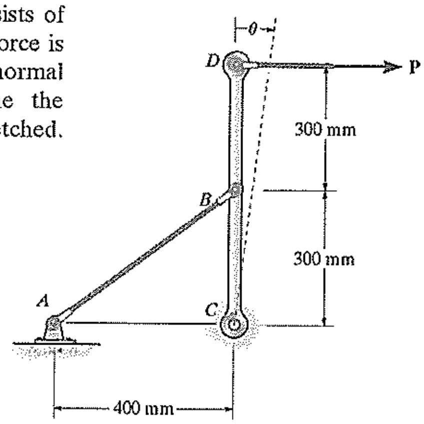
$$= 501.255 \text{ mm}$$



$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm} \qquad \textbf{Ans}$$

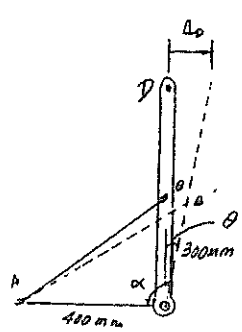
2–9. Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB} AB$$

$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$



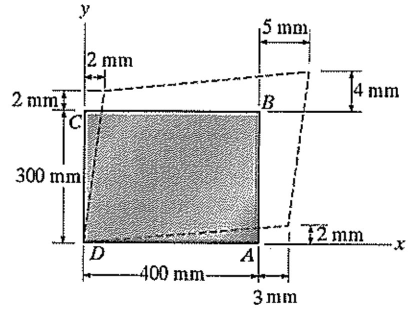
$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

$$\alpha = 90.4185°$$

$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

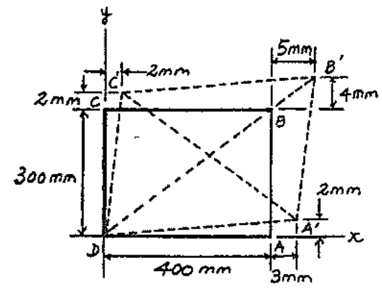
$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180°}\right)(0.4185) = 4.38 \text{ mm} \qquad \textbf{Ans}$$

2–14. The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals AC and DB.



Geometry :

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$
$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$
$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$

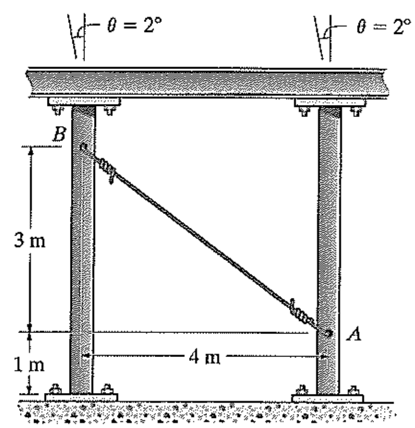


Average Normal Strain :

$$\varepsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$
$$= 0.00160 \text{ mm/mm} = 1.60(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$
$$= 0.0128 \text{ mm/mm} = 12.8(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans}$$

2–15. The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.

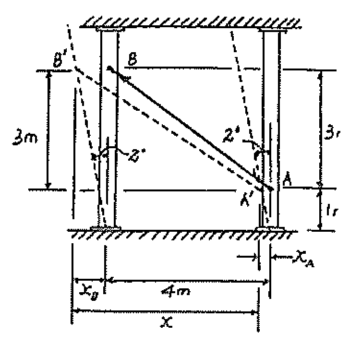


Geometry : The vertical displacement is negligible.

$$x_A = (1)\left(\frac{2°}{180°}\right)\pi = 0.03491 \text{ m}$$
$$x_B = (4)\left(\frac{2°}{180°}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

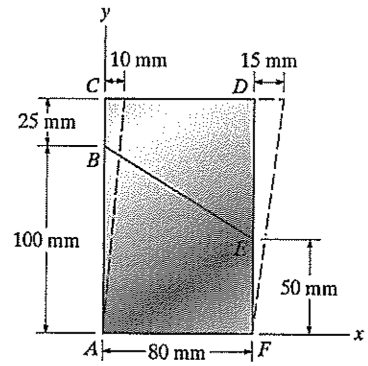
$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$
$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$



Average Normal Stress :

$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$
$$= \frac{5.08416 - 5}{5} = 16.8(10^{-3}) \text{ m/m} \qquad \text{Ans}$$

2–27. The material distorts into the dashed position shown. Determine (a) the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and (b) the average normal strain along line BE.



Since there is no deformation occuring along the y and x axes.

$\epsilon_x = 0$ Ans.

$$\epsilon_y = \frac{\sqrt{(125)^2 + (10)^2} - 125}{125} = 0.00319 \quad \text{Ans.}$$

$$\tan \gamma_{xy} = \frac{10}{125}$$

$$\gamma_{xy} = 0.0798 \text{ rad} \quad \text{Ans}$$

From geometry :

$$\frac{BB'}{100} = \frac{10}{125}; \qquad BB' = 8 \text{ mm}$$

$$\frac{EE'}{50} = \frac{15}{125}; \qquad EE' = 6 \text{ mm}$$

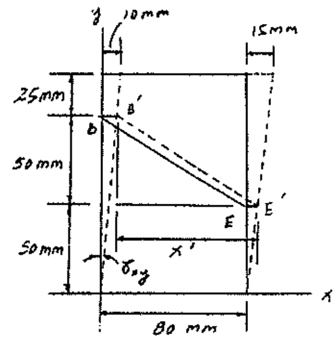
$$BE = \sqrt{50^2 + 80^2} = \sqrt{8900} \text{ mm}$$

$$x' = 80 + EE' - BB' = 80 + 6 - 8 = 78 \text{ mm}$$

$$B'E' = \sqrt{50^2 + 78^2} = \sqrt{8584} \text{ mm}$$

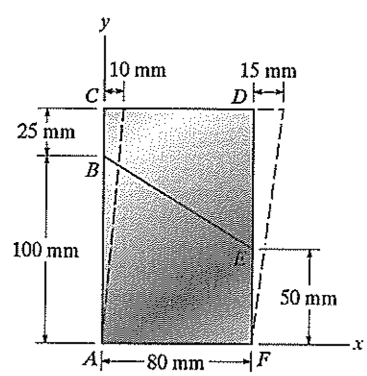
$$\epsilon_{BE} = \frac{B'E' - BE}{BE} = \frac{\sqrt{8584} - \sqrt{8900}}{\sqrt{8900}}$$

$$= -0.0179 \text{ mm/mm} \quad \text{Ans}$$



Negative sign indicates shortening of BE.

***2–28.** The material distorts into the dashed position shown. Determine the average normal strain that occurs along the diagonals AD and CF.



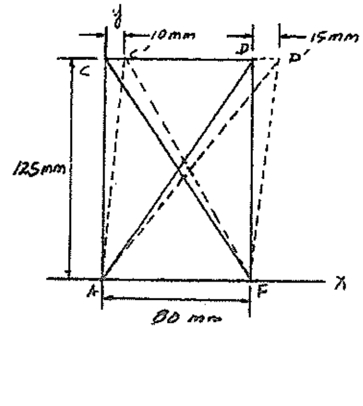
$$AD = CF = \sqrt{(80)^2 + (125)^2} = \sqrt{22025} \text{ mm}$$

$$C'F = \sqrt{(70)^2 + (125)^2} = \sqrt{20525} \text{ mm}$$

$$AD' = \sqrt{(95)^2 + (125)^2} = \sqrt{24650} \text{ mm}$$

$$\epsilon_{AD} = \frac{AD' - AD}{AD} = \frac{\sqrt{24650} - \sqrt{22025}}{\sqrt{22025}}$$

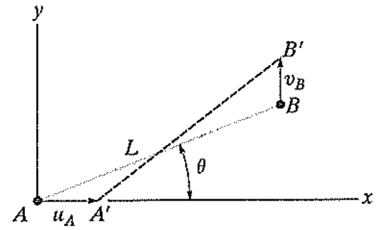
$$= 0.0579 \text{ mm/mm} \quad \text{Ans}$$



$$\epsilon_{CF} = \frac{C'F - CF}{CF} = \frac{\sqrt{20525} - \sqrt{22025}}{\sqrt{22025}}$$

$$= -0.0347 \text{ mm/mm} \quad \text{Ans}$$

2–34. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.

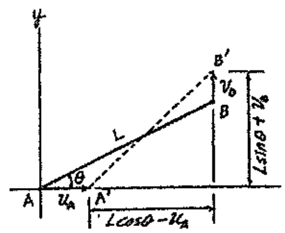


Geometry :

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$
$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal strain :

$$\varepsilon_{AB} = \frac{L_{A'B'} - L}{L}$$
$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$



Neglecting higher terms u_A^2 and v_B^2

$$\varepsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem :

$$\varepsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$
$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L} \qquad \textbf{Ans}$$

2–35. If the normal strain is defined in reference to the final length, that is,

$$\epsilon_n' = \lim_{p\to p'}\left(\frac{\Delta s' - \Delta s}{\Delta s'}\right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon_n' = \epsilon_n\epsilon_n'$.

$$\varepsilon_n = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\varepsilon_n - \varepsilon_n' = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$
$$= \frac{\Delta S'^2 - \Delta S\Delta S' - \Delta S'\Delta S + \Delta S^2}{\Delta S\Delta S'}$$
$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S'\Delta S}{\Delta S\Delta S'}$$
$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S\Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S}\right)\left(\frac{\Delta S' - \Delta S}{\Delta S'}\right)$$
$$= \varepsilon_n\varepsilon_n' \quad (Q.E.D)$$