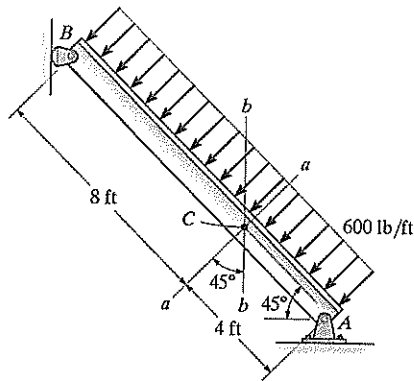
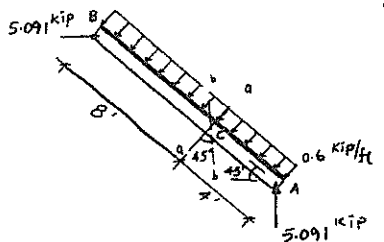


*1-12. Determine the resultant internal loadings acting on (a) section $a-a$ and (b) section $b-b$. Each section is located through the centroid, point C .



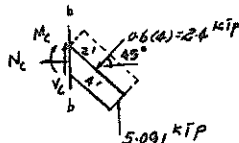
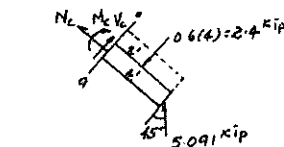
(a)

$$\begin{aligned} \uparrow + \Sigma F_x = 0; & \quad N_C + 5.091 \sin 45^\circ = 0 \\ & \quad N_C = -3.60 \text{ kip} \quad \text{Ans} \\ \rightarrow + \Sigma F_y = 0; & \quad V_C + 5.091 \cos 45^\circ - 2.4 = 0 \\ & \quad V_C = -1.20 \text{ kip} \quad \text{Ans} \\ \curvearrowright + \Sigma M_C = 0; & \quad -M_C - 2.4(2) + 5.091 \cos 45^\circ(4) = 0 \\ & \quad M_C = 9.60 \text{ kip} \cdot \text{ft} \quad \text{Ans} \end{aligned}$$

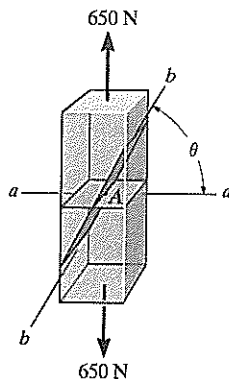


(b)

$$\begin{aligned} \leftarrow + \Sigma F_x = 0; & \quad N_C + 2.4 \cos 45^\circ = 0 \\ & \quad N_C = -1.70 \text{ kip} \quad \text{Ans} \\ + \uparrow \Sigma F_y = 0; & \quad V_C + 5.091 - 2.4 \sin 45^\circ = 0 \\ & \quad V_C = -3.39 \text{ kip} \quad \text{Ans} \\ \curvearrowright + \Sigma M_C = 0; & \quad -M_C - 2.4(2) + 5.091 \cos 45^\circ(4) = 0 \\ & \quad M_C = 9.60 \text{ kip} \cdot \text{ft} \quad \text{Ans} \end{aligned}$$



1-13. Determine the resultant internal normal and shear forces in the member at (a) section $a-a$ and (b) section $b-b$, each of which passes through point A . Take $\theta = 60^\circ$. The 650-N load is applied along the centroidal axis of the member.



Equations of Equilibrium: For section $a-a$

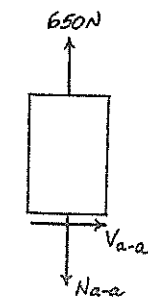
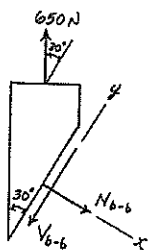
$$\begin{aligned} + \uparrow \Sigma F_y = 0; & \quad 650 - N_{a-a} = 0 \\ & \quad N_{a-a} = 650 \text{ N} \quad \text{Ans} \end{aligned}$$

$$\rightarrow \Sigma F_x = 0; \quad V_{a-a} = 0 \quad \text{Ans}$$

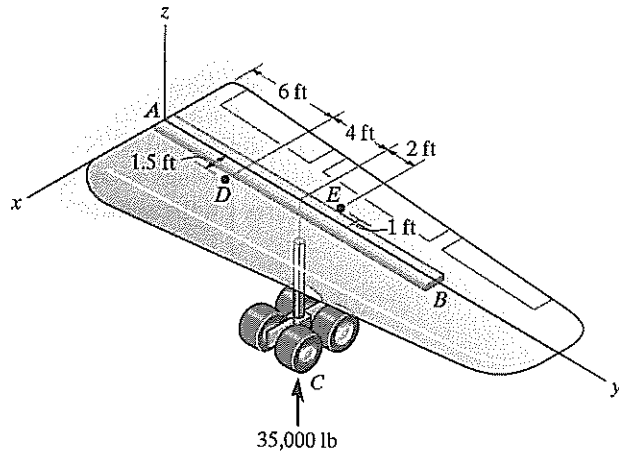
Equations of Equilibrium: For section $b-b$

$$\begin{aligned} \nearrow \Sigma F_y = 0; & \quad 650 \cos 30^\circ - V_{b-b} = 0 \\ & \quad V_{b-b} = 563 \text{ N} \quad \text{Ans} \end{aligned}$$

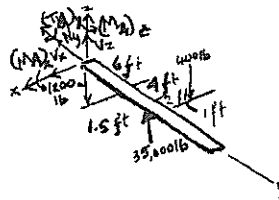
$$\begin{aligned} + \Sigma F_x = 0; & \quad N_{b-b} - 650 \sin 30^\circ = 0 \\ & \quad N_{b-b} = 325 \text{ N} \quad \text{Ans} \end{aligned}$$



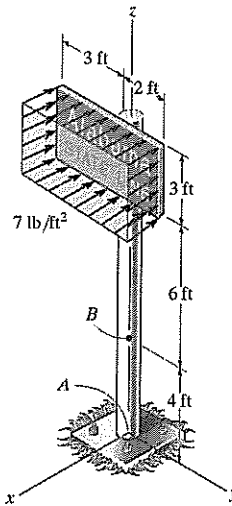
*1-24. The main beam AB supports the load on the wing of the airplane. The loads consist of the wheel reaction of 35,000 lb at C , the 1200-lb weight of fuel in the tank of the wing, having a center of gravity at D , and the 400-lb weight of the wing, having a center of gravity at E . If it is fixed to the fuselage at A , determine the resultant internal loadings on the beam at this point. Assume that the wing does not transfer any of the loads to the fuselage, except through the beam.



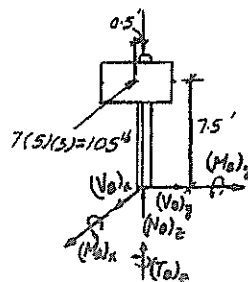
$$\begin{aligned} \Sigma F_x = 0; \quad (V_A)_x &= 0 && \text{Ans} \\ \Sigma F_y = 0; \quad (N_A)_y &= 0 && \text{Ans} \\ \Sigma F_z = 0; \quad (V_A)_z - 1200 - 400 + 35000 &= 0 \\ (V_A)_z &= -33.4 \text{ kip} && \text{Ans} \\ \Sigma M_x = 0; \quad (M_A)_x - 1200(6) + 35000(10) - 400(12) &= 0 \\ (M_A)_x &= 338 \text{ kip} \cdot \text{ft} && \text{Ans} \\ \Sigma M_y = 0; \quad (T_A)_y + 1200(1.5) - 400(1) &= 0 \\ (T_A)_y &= -1.40 \text{ kip} \cdot \text{ft} && \text{Ans} \\ \Sigma M_z = 0; \quad (M_A)_z &= 0 && \text{Ans} \end{aligned}$$



1-25. Determine the resultant internal loadings acting on the cross section through point B of the signpost. The post is fixed to the ground and a uniform pressure of 7 lb/ft^2 acts perpendicular to the face of the sign.



$$\begin{aligned} \Sigma F_x = 0; \quad (V_B)_x - 105 &= 0; \quad (V_B)_x = 105 \text{ lb} && \text{Ans} \\ \Sigma F_y = 0; \quad (V_B)_y &= 0 && \text{Ans} \\ \Sigma F_z = 0; \quad (N_B)_z &= 0 && \text{Ans} \\ \Sigma M_x = 0; \quad (M_B)_x &= 0 && \text{Ans} \\ \Sigma M_y = 0; \quad (M_B)_y - 105(7.5) &= 0; \quad (M_B)_y = 788 \text{ lb} \cdot \text{ft} && \text{Ans} \\ \Sigma M_z = 0; \quad (T_B)_z - 105(0.5) &= 0; \quad (T_B)_z = 52.5 \text{ lb} \cdot \text{ft} && \text{Ans} \end{aligned}$$



1-30. The pipe has a mass of 12 kg/m. If it is fixed to the wall at A, determine the resultant internal loadings acting on the cross section through B.

Equations of Equilibrium: For point B

$$\Sigma F_x = 0; \quad (V_B)_x = 0 \quad \text{Ans}$$

$$\Sigma F_y = 0; \quad (N_B)_y + \frac{4}{5}(750) = 0$$

$$(N_B)_y = -600 \text{ N} \quad \text{Ans}$$

$$\Sigma F_z = 0; \quad (V_B)_z - 235.44 - 235.44 - \frac{3}{5}(750) = 0$$

$$(V_B)_z = 921 \text{ N} \quad \text{Ans}$$

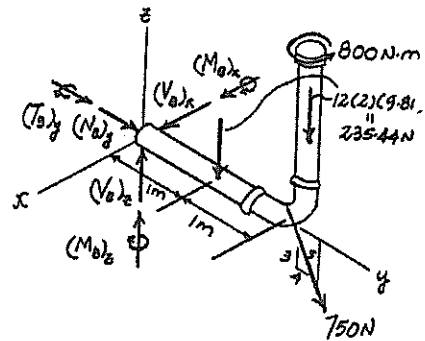
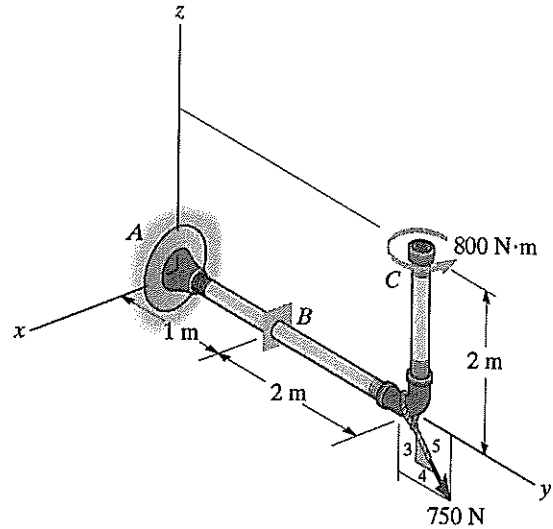
$$\Sigma M_x = 0; \quad (M_B)_x - 235.44(1) - 235.44(2) - \frac{3}{5}(750)(2) = 0$$

$$(M_B)_x = 1606 \text{ N}\cdot\text{m} \quad \text{Ans}$$

$$\Sigma M_y = 0; \quad (T_B)_y = 0 \quad \text{Ans}$$

$$\Sigma M_z = 0; \quad (M_B)_z + 800 = 0$$

$$(M_B)_z = -800 \text{ N}\cdot\text{m} \quad \text{Ans}$$



1-31. The curved rod has a radius r and is fixed to the wall at B. Determine the resultant internal loadings acting on the cross section through A which is located at an angle θ from the horizontal.

Equations of Equilibrium: For point A

$$+\Sigma F_x = 0; \quad P \cos \theta - N_A = 0$$

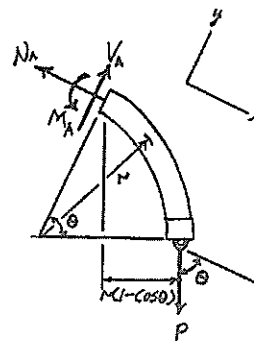
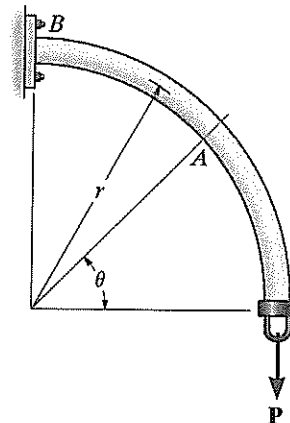
$$N_A = P \cos \theta \quad \text{Ans}$$

$$+\Sigma F_y = 0; \quad V_A - P \sin \theta = 0$$

$$V_A = P \sin \theta \quad \text{Ans}$$

$$+\Sigma M_A = 0; \quad M_A - P[r(1 - \cos \theta)] = 0$$

$$M_A = Pr(1 - \cos \theta) \quad \text{Ans}$$



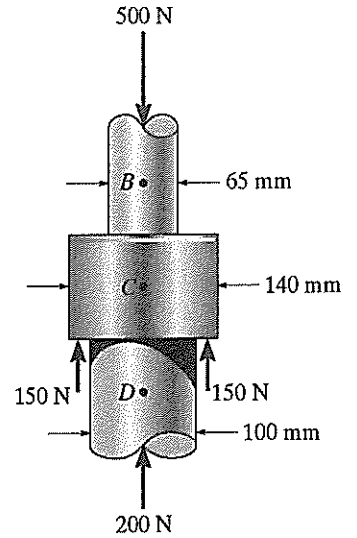
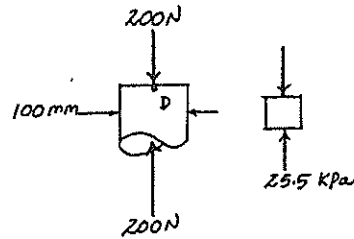
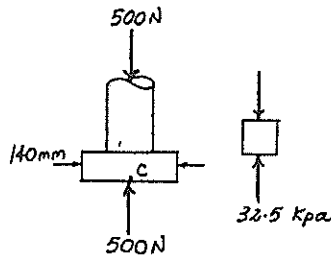
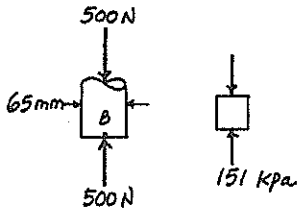
1-37. The thrust bearing is subjected to the loads shown. Determine the average normal stress developed on cross sections through points *B*, *C*, and *D*. Sketch the results on a differential volume element located at each section.

Average Normal Stress :

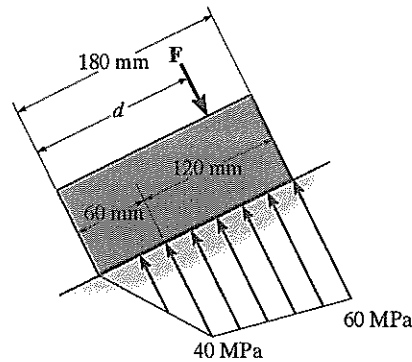
$$\sigma_B = \frac{500}{\frac{\pi}{4}(65)^2} = 151 \text{ kPa} \quad \text{Ans}$$

$$\sigma_C = \frac{500}{\frac{\pi}{4}(140)^2} = 32.5 \text{ kPa} \quad \text{Ans}$$

$$\sigma_D = \frac{200}{\frac{\pi}{4}(100)^2} = 25.5 \text{ kPa} \quad \text{Ans}$$



1-38. The small block has a thickness of 5 mm. If the stress distribution at the support developed by the load varies as shown, determine the force *F* applied to the block, and the distance *d* to where it is applied.



$$F = \int \sigma dA = \text{volume under stress diagram}$$

$$F = \frac{1}{2}(0.06)(40)(10^6)(0.005) + (0.120)(40)(10^6)(0.005) + \frac{1}{2}(0.120)(20)(10^6)(0.005)$$

$$F = 36 \text{ kN} \quad \text{Ans}$$

Require

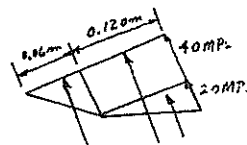
$$Fd = \int x(\sigma dA)$$

$$36.0(10^3)d = \frac{2}{3}(0.06)(\frac{1}{2})(0.06)(40)(10^6)(0.005) + (0.06 + \frac{1}{2}(0.120))(0.120)(40)(10^6)(0.005) +$$

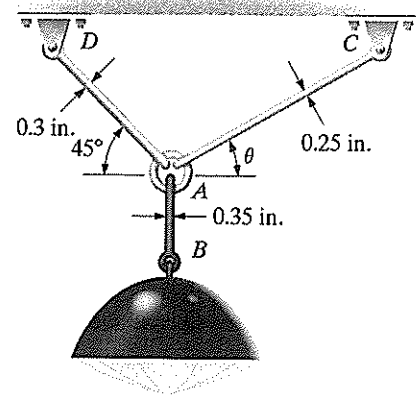
$$(0.06 + \frac{2}{3}(0.120))(\frac{1}{2})(0.120)(20)(10^6)(0.005)$$

$$36.0(10^3)d = 3960$$

$$d = 0.110 = 110 \text{ mm} \quad \text{Ans}$$

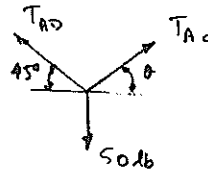


*1-44. The 50-lb lamp is supported by three steel rods connected by a ring at A . Determine the angle of orientation θ of AC such that the average normal stress in rod AC is twice the average normal stress in rod AD . What is the magnitude of stress in each rod? The diameter of each rod is given in the figure.



$$\sigma_{AD} = \frac{T_{AD}}{\frac{\pi}{4}(0.3)^2}; \quad T_{AD} = (0.070686)\sigma_{AD}$$

$$\sigma_{AC} = 2\sigma_{AD} = \frac{T_{AC}}{\frac{\pi}{4}(0.25)^2}; \quad T_{AC} = (0.098175)\sigma_{AD}$$



$$\rightarrow \Sigma F_x = 0; \quad -T_{AD} \cos 45^\circ + T_{AC} \cos \theta = 0 \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad T_{AC} \sin \theta + T_{AD} \sin 45^\circ - 50 = 0 \quad (2)$$

Thus

$$-(0.070686)\sigma_{AD}(\cos 45^\circ) + (0.098175)\sigma_{AD}(\cos \theta) = 0$$

$$\theta = 59.39^\circ = 59.4^\circ \quad \text{Ans}$$

From Eq. (2) :

$$(0.098175)\sigma_{AD} \sin 59.39^\circ + (0.070686)\sigma_{AD} \sin 45^\circ - 50 = 0$$

$$\sigma_{AD} = 371.8 \text{ psi} = 372 \text{ psi} \quad \text{Ans}$$

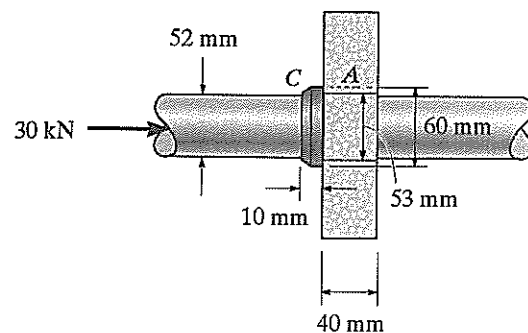
Hence,

$$\sigma_{AC} = 2(371.8) = 744 \text{ psi} \quad \text{Ans}$$

And,

$$\sigma_{AB} = \frac{T_{AB}}{\frac{\pi}{4}(0.35)^2} = \frac{50}{\frac{\pi}{4}(0.35)^2} = 520 \text{ psi} \quad \text{Ans}$$

1-45. The shaft is subjected to the axial force of 30 kN. If the shaft passes through the 53-mm diameter hole in the fixed support A , determine the bearing stress acting on the collar C . Also, what is the average shear stress acting along the inside surface of the collar where it is fixed connected to the 52-mm diameter shaft?



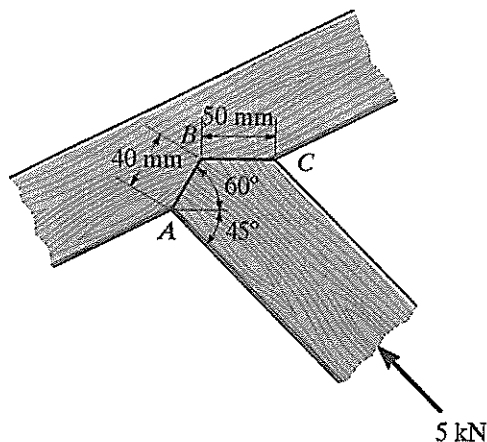
Bearing Stress :

$$\sigma_b = \frac{P}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.06^2 - 0.053^2)} = 48.3 \text{ MPa} \quad \text{Ans}$$

Average Shear Stress :

$$\tau_{avg} = \frac{V}{A} = \frac{30(10^3)}{\pi(0.052)(0.01)} = 18.4 \text{ MPa} \quad \text{Ans}$$

*1-52. The joint is subjected to the axial member force of 5 kN. Determine the average normal stress acting on sections AB and BC. Assume the member is smooth and is 50-mm thick.



$$\rightarrow \Sigma F_x = 0; \quad N_{BA} \cos 30^\circ - 5 \cos 45^\circ = 0$$

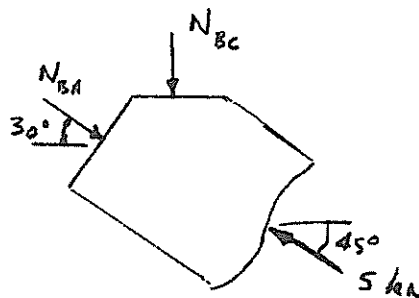
$$N_{BA} = 4.082 \text{ kN}$$

$$+ \uparrow \Sigma F_y = 0; \quad -N_{BC} - 4.082 \sin 30^\circ + 5 \sin 45^\circ = 0$$

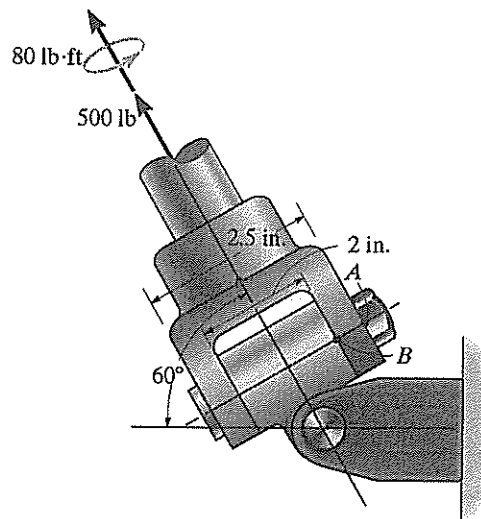
$$N_{BC} = 1.494 \text{ kN}$$

$$\sigma_{BA} = \frac{N_{BA}}{A_{BA}} = \frac{4.082(10^3)}{(0.04)(0.05)} = 2.04 \text{ MPa} \quad \text{Ans}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{1.494(10^3)}{(0.05)(0.05)} = 0.598 \text{ MPa} \quad \text{Ans}$$



1-53. The yoke is subjected to the force and couple moment. Determine the average shear stress in the bolt acting on the cross sections through A and B. The bolt has a diameter of 0.25 in. *Hint:* The couple moment is resisted by a set of couple forces developed in the shank of the bolt.



At A force on bolt shank is zero, then

$$\tau_A = 0 \quad \text{Ans}$$

Equations of Equilibrium: Force on bolt shank at B.

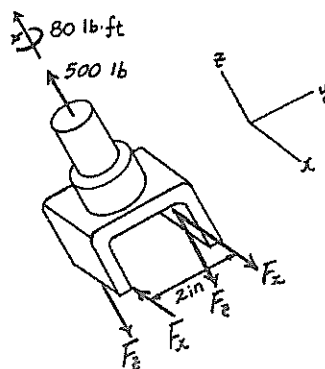
$$\Sigma F_z = 0; \quad 500 - 2F_z = 0 \quad F_z = 250 \text{ lb}$$

$$\Sigma M_z = 0; \quad 80 \text{ lb} \cdot \text{ft} \left(\frac{12 \text{ in}}{\text{ft}} \right) - F_z (2 \text{ in.}) = 0$$

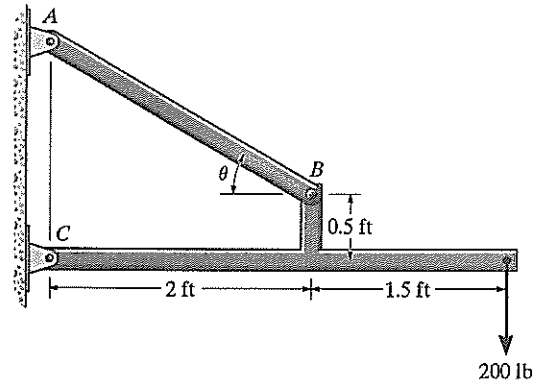
$$F_z = 480 \text{ lb}$$

Average Shear Stress: The bolt shank subjected to a shear force of $V_B = F_B = \sqrt{250^2 + 480^2} = 541.2 \text{ lb}$.

$$(\tau_B)_{\text{avg}} = \frac{541.2}{\frac{\pi}{4}(0.25)^2} = 11.0 \text{ ksi} \quad \text{Ans}$$



1-69. The frame is subjected to the load of 200 lb. Determine the average shear stress in the bolt at A as a function of the bar angle θ . Plot this function, $0 \leq \theta \leq 90^\circ$, and indicate the values of θ for which this stress is a minimum. The bolt has a diameter of 0.25 in. and is subjected to single shear.



Support Reactions :

$$(+\Sigma M_C = 0; \quad F_{AB} \cos \theta (0.5) + F_{AB} \sin \theta (2) - 200(3.5) = 0$$

$$F_{AB} (0.5 \cos \theta + 2 \sin \theta) = 700$$

$$F_{AB} = \frac{700}{0.5 \cos \theta + 2 \sin \theta}$$

Average Shear Stress : Pin A is subjected to single shear. Hence, $V_A = F_{AB}$

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{\left(\frac{700}{0.5 \cos \theta + 2 \sin \theta} \right)}{\frac{\pi}{4} (0.25)^2}$$

$$= \left\{ \frac{14260}{0.5 \cos \theta + 2 \sin \theta} \right\} \text{ psi}$$

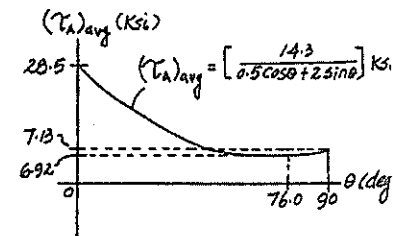
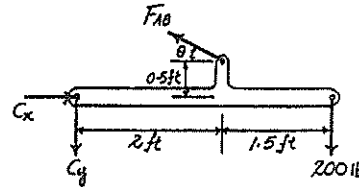
$$= \left\{ \frac{14.3}{0.5 \cos \theta + 2 \sin \theta} \right\} \text{ ksi} \quad \text{Ans}$$

$$\frac{d\tau}{d\theta} = 0$$

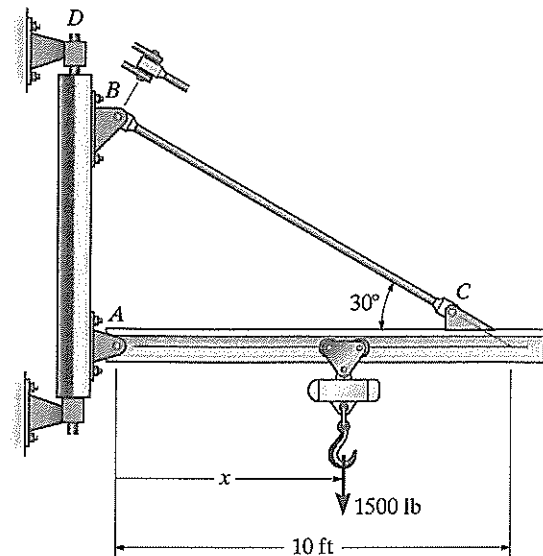
$$\frac{(0.5 \cos \theta + 2 \sin \theta)(0) - (-0.5 \sin \theta + 2 \cos \theta)(14260)}{(0.5 \cos \theta + 2 \sin \theta)^2} = 0$$

$$0.5 \sin \theta - 2 \cos \theta = 0$$

$$\tan \theta = 4; \quad \theta_{min} = 76.0^\circ \quad \text{Ans}$$



1-70. The jib crane is pinned at A and supports a chain hoist that can travel along the bottom flange of the beam, $1 \text{ ft} \leq x \leq 12 \text{ ft}$. If the hoist is rated to support a maximum of 1500 lb, determine the maximum average normal stress in the $\frac{3}{4}$ -in.-diameter tie rod BC and the maximum average shear stress in the $\frac{5}{8}$ -in.-diameter pin at B.



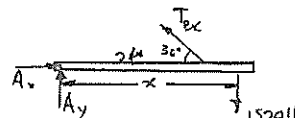
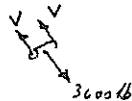
$$(+\Sigma M_A = 0; \quad T_{BC} \sin 30^\circ (10) - 1500(x) = 0$$

Maximum T_{BC} occurs when $x = 12 \text{ ft}$

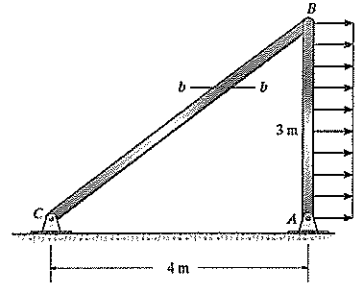
$$T_{BC} = 3600 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{3600}{\frac{\pi}{4} (0.75)^2} = 8.15 \text{ ksi} \quad \text{Ans}$$

$$\tau = \frac{V}{A} = \frac{3600/2}{\frac{\pi}{4} (5/8)^2} = 5.87 \text{ ksi} \quad \text{Ans}$$



*1-76. The two-member frame is subjected to the distributed loading shown. Determine the largest intensity w of the uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section $b-b$ to exceed $\sigma = 15 \text{ MPa}$ and $\tau = 16 \text{ MPa}$, respectively. Member CB has a square cross-section of 30 mm on each side.



Support Reactions : FBD (a)

$$\left(+ \Sigma M_A = 0; \quad \frac{4}{5} F_{BC}(3) - 3w(1.5) = 0 \quad F_{BC} = 1.875w \right)$$

Equations of Equilibrium : For section $b-b$, FBD (b)

$$\rightarrow \Sigma F_x = 0; \quad \frac{4}{5}(1.875w) - V_{b-b} = 0 \quad V_{b-b} = 1.50w$$

$$+ \uparrow \Sigma F_y = 0; \quad \frac{3}{5}(1.875w) - N_{b-b} = 0 \quad N_{b-b} = 1.125w$$

Average Normal Stress And Shear Stress : The cross-sectional area of section $b-b$, $A' = \frac{SA}{3}$; where $A = (0.03)(0.03) = 0.90(10^{-3}) \text{ m}^2$.

$$\text{Then } A' = \frac{5}{3}(0.90)(10^{-3}) = 1.50(10^{-3}) \text{ m}^2.$$

Assume failure due to normal stress.

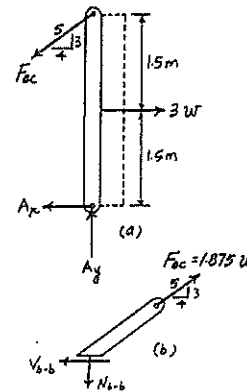
$$(\sigma_{b-b})_{\text{Allow}} = \frac{N_{b-b}}{A'}; \quad 15(10^6) = \frac{1.125w}{1.50(10^{-3})}$$

$$w = 20000 \text{ N/m} = 20.0 \text{ kN/m}$$

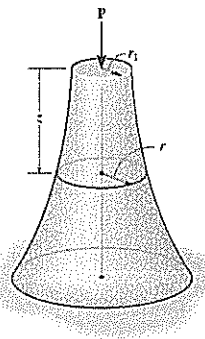
Assume failure due to shear stress.

$$(\tau_{b-b})_{\text{Allow}} = \frac{V_{b-b}}{A'}; \quad 16(10^6) = \frac{1.50w}{1.50(10^{-3})}$$

$$w = 16000 \text{ N/m} = 16.0 \text{ kN/m} \quad (\text{Controls !}) \quad \text{Ans}$$



1-77. The pedestal supports a load P at its center. If the material has a mass density ρ , determine the radial dimension r as a function of z so that the average normal stress in the pedestal remains constant. The cross section is circular.



Require :

$$\sigma = \frac{P + W_1}{A} = \frac{P + W_1 + dW}{A + dA}$$

$$P dA + W_1 dA = A dW$$

$$\frac{dW}{dA} = \frac{P + W_1}{A} = \sigma \quad (1)$$

$$dA = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr$$

$$dW = \pi r^2(\rho g) dz$$

From Eq. (1).

$$\frac{\pi r^2(\rho g) dz}{2\pi r dr} = \sigma$$

$$\frac{r \rho g dz}{2 dr} = \sigma$$

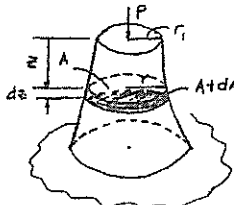
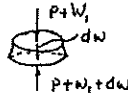
$$\frac{\rho g}{2\sigma} \int_0^z dz = \int_{r_1}^r \frac{dr}{r}$$

$$\frac{\rho g z}{2\sigma} = \ln \frac{r}{r_1}; \quad r = r_1 e^{\left(\frac{\rho g}{2\sigma}\right)z}$$

$$\text{However, } \sigma = \frac{P}{\pi r_1^2}$$

$$r = r_1 e^{\left(\frac{\rho g}{2P/\pi r_1^2}\right)z}$$

Ans



1-87. The frame is subjected to the load of 1.5 kip. Determine the required diameter of the pins at A and B if the allowable shear stress for the material is $\tau_{\text{allow}} = 6$ ksi. Pin A is subjected to double shear, whereas pin B is subjected to single shear.

Support Reactions : From FBD (a),

$$\begin{aligned} +\Sigma M_D = 0; & F_{BC}(\sin 45^\circ)(5) - 1.5(7) = 0 \\ & F_{BC} = 2.970 \text{ kip} \end{aligned}$$

From FBD (b),

$$+\Sigma M_A = 0; D_y(10) - 1.5(7) = 0 \quad D_y = 1.05 \text{ kip}$$

$$\leftarrow \Sigma F_x = 0; A_x - 1.5 = 0 \quad A_x = 1.50 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; 1.05 - A_y = 0 \quad A_y = 1.05 \text{ kip}$$

Allowable Shear Stress : Design of pin sizes

For pin A

Pin A is subjected to double shear and

$$F_A = \sqrt{1.50^2 + 1.05^2} = 1.831 \text{ kip.}$$

$$\text{Therefore, } V_A = \frac{F_A}{2} = 0.9155 \text{ kip}$$

$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \quad 6 = \frac{0.9155}{\frac{\pi}{4}d_A^2}$$

$$d_A = 0.441 \text{ in.} \quad \text{Ans}$$

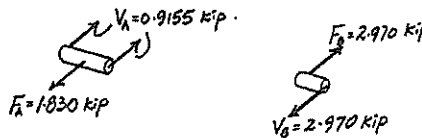
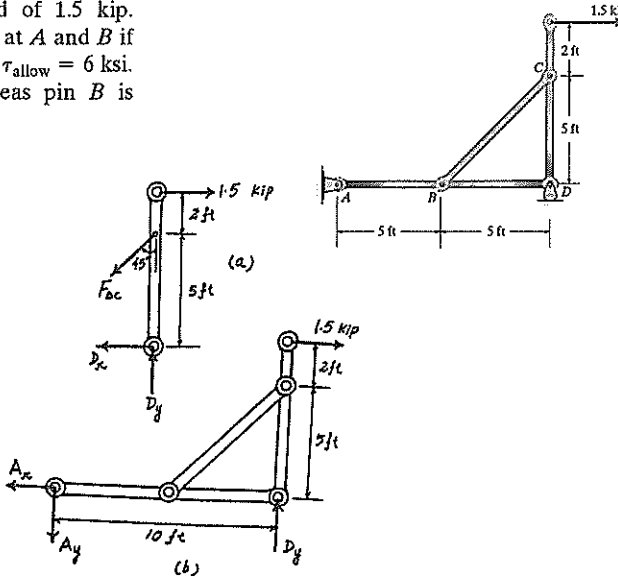
For pin B

Pin B is subjected to single shear. Therefore,

$$V_B = F_B = F_{BC} = 2.970 \text{ kip}$$

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 6 = \frac{2.970}{\frac{\pi}{4}d_B^2}$$

$$d_B = 0.794 \text{ in.} \quad \text{Ans}$$



*1-88. The two steel wires AB and AC are used to support the load. If both wires have an allowable tensile stress of $\sigma_{\text{allow}} = 200$ MPa, determine the required diameter of each wire if the applied load is $P = 5$ kN.

$$+\Sigma F_x = 0; \quad \frac{4}{5}F_{AC} - F_{AB} \sin 60^\circ = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AC} + F_{AB} \cos 60^\circ - 5 = 0 \quad (2)$$

Solving Eqs. (1) and (2) yields :

$$F_{AB} = 4.3496 \text{ kN}; \quad F_{AC} = 4.7086 \text{ kN}$$

$$\text{Applying } \sigma_{\text{allow}} = \frac{P}{A}$$

For wire AB,

$$200(10^6) = \frac{4.3496(10^3)}{\frac{\pi}{4}(d_{AB})^2}$$

$$d_{AB} = 0.00526 \text{ m} = 5.26 \text{ mm} \quad \text{Ans}$$

For wire AC,

$$200(10^6) = \frac{4.7086(10^3)}{\frac{\pi}{4}(d_{AC})^2}$$

$$d_{AC} = 0.00548 \text{ m} = 5.48 \text{ mm} \quad \text{Ans}$$

