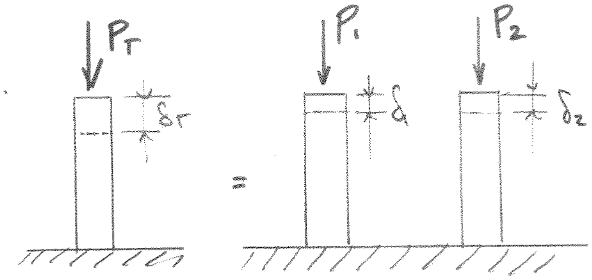
CLASS #9 - INDETERMINATE AXIAL MEMBERS

OBJECTIVES:
1. REVIEW PRINCIPLE of SUPERPOSITION
2. STATICALLY INDETERMINATE AXIAL MEMBERS
3. DETAIL FORCE METHOD
4. THERMAL EFFECTS

① SUPERPOSITION - IF A SYSTEM IS LINEAR ($\sigma = E\epsilon$), THEN CAN EMPLOY PRINCIPLE of SUPERPOSITION.

CAN DIVIDE LOADING INTO SMALLER COMPONENTS & SUM THE DEFORMATION FROM EACH

e.g.

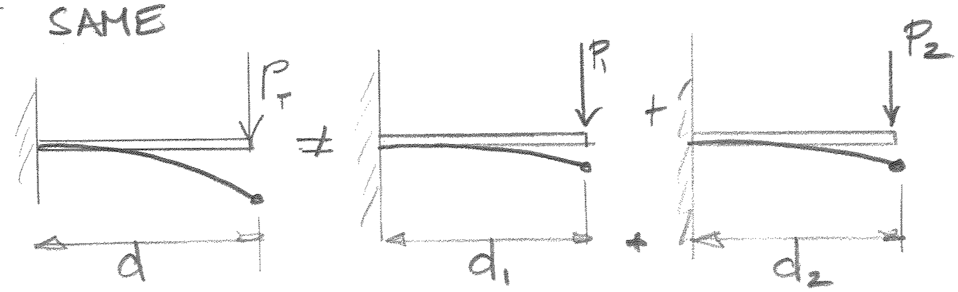


$$P_T = P_1 + P_2$$

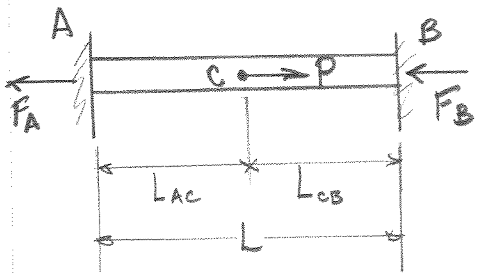
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\delta_T = \delta_1 + \delta_2$$

NOTE: GEOMETRY MUST REMAIN NEARLY THE SAME



② STATICALLY INDETERMINATE MEMBERS



2 UNKNOWNS : F_A, F_B

1 EQUATION : $\sum F = -F_A - F_B + P = 0$
(EQUILIBRIUM)

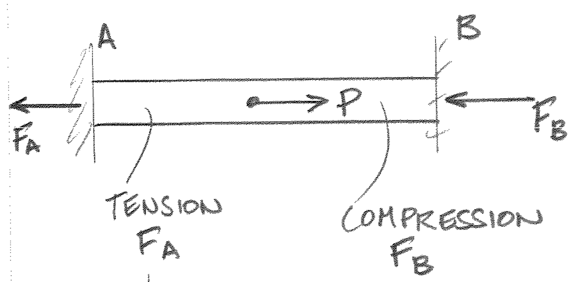
∴ STATICALLY INDETERMINATE!

BUT, CAN STILL SOLVE if WE FIND ONE MORE EQUATION:

$\delta_{AB} = 0$ (COMPATIBILITY or KINEMATIC CONDITION)

$\downarrow$

ANY CONDITION on DISPLACEMENT



$\delta_{AC} = \dfrac{F_A L_{AC}}{AE}$ $\qquad$ $\delta_{CB} = \dfrac{-F_B L_{CB}}{AE}$ $\longrightarrow$ $\delta_{AB} = 0 = \dfrac{F_A L_{AC}}{AE} - \dfrac{F_B L_{CB}}{AE}$

$$0 = \frac{F_A L_{AC} - F_B L_{CB}}{AE}$$

TWO EQUATIONS: $F_A L_{AC} - F_B L_{CB} = 0$

$$F_A + F_B = P$$

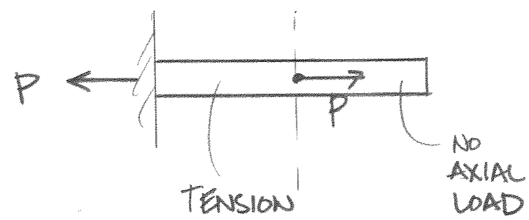
$$\downarrow$$

$$F_A = P\left(\frac{L_{CB}}{L}\right) \quad \& \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

③ FORCE METHOD

WHILE ABOVE PROCEDURE WORKS, CAN SIMPLIFY ANALYSIS. USE "FORCE METHOD" WHICH EMPLOYS SUPERPOSITION.

STEP 1: ① REMOVE REDUNDANT REACTION

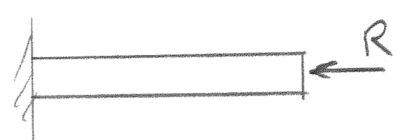


$$\delta_{AC} = \delta_{AB} = \frac{P L_{AC}}{EA}$$

STEP 2: DETERMINE RESPONSE at REACTION

$$\delta_{AB} = \frac{P L_{AC}}{EA}$$

STEP 3: ENFORCE COMPABILITY $(\delta_{AB} = 0)$

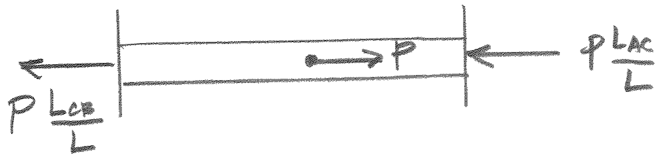


$$\delta_{AB} = \frac{R L_{AB}}{EA}$$

$$\frac{P L_{AC}}{EA} = \frac{R L_{AB}}{EA}$$

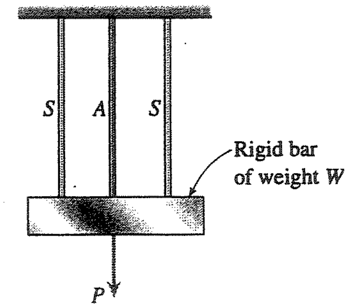
$$\therefore \quad R = \frac{P L_{AC}}{L}$$

STEP 4: APPLY EQUILIBRIUM



Problem 2.4-10 A rigid bar of weight $W = 800$ N hangs from three equally spaced vertical wires, two of steel and one of aluminum (see figure). The wires also support a load P acting at the midpoint of the bar. The diameter of the steel wires is 2 mm, and the diameter of the aluminum wire is 4 mm.

What load P_{allow} can be supported if the allowable stress in the steel wires is 220 MPa and in the aluminum wire is 80 MPa? (Assume $E_s = 210$ GPa and $E_a = 70$ GPa.)



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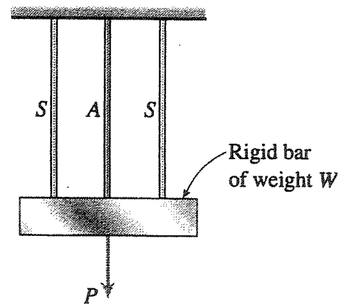
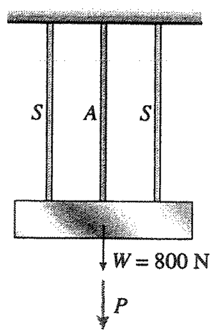


Solution 2.4-10 **Rigid bar hanging from three wires**



STEEL WIRES

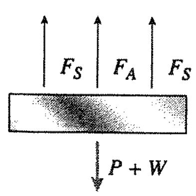
$d_s = 2$ mm $\quad \sigma_s = 220$ MPa $\quad E_s = 210$ GPa

ALUMINUM WIRES

$d_A = 4$ mm $\quad \sigma_A = 80$ MPa

$E_A = 70$ GPa

FREE-BODY DIAGRAM OF RIGID BAR



EQUATION OF EQUILIBRIUM

$\Sigma F_{vert} = 0$

$$2F_s + F_A - P - W = 0 \qquad \text{(Eq. 1)}$$

EQUATION OF COMPATIBILITY

$$\delta_s = \delta_A \qquad \text{(Eq. 2)}$$

FORCE DISPLACEMENT RELATIONS

$$\delta_s = \frac{F_s L}{E_s A_s} \qquad \delta_A = \frac{F_A L}{E_A A_A} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{F_s L}{E_s A_s} = \frac{F_A L}{E_A A_A} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$F_A = (P + W)\left(\frac{E_A A_A}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 6)}$$

$$F_s = (P + W)\left(\frac{E_s A_s}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 7)}$$

STRESSES IN THE WIRES

$$\sigma_A = \frac{F_A}{A_A} = \frac{(P + W)E_A}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 8)}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{(P + W)E_s}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 9)}$$

ALLOWABLE LOADS (FROM EQS. (8) AND (9))

$$P_A = \frac{\sigma_A}{E_A}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 10)}$$

$$P_s = \frac{\sigma_s}{E_s}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 11)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQS. (10) AND (11):

$$A_s = \frac{\pi}{4}(2 \text{ mm})^2 = 3.1416 \text{ mm}^2$$

$$A_A = \frac{\pi}{4}(4 \text{ mm})^2 = 12.5664 \text{ mm}^2$$

$P_A = 1713$ N

$P_s = 1504$ N

Steel governs. $\quad P_{allow} = 1500$ N $\longleftarrow$