CLASS 13 – BEHAVIOR OF SHAFTS & RODS

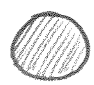
OBJECTIVES: ① DESCRIBE BEHAVIOR of THIN WALLED SECTIONS
② UNDERSTAND INELASTIC TORSION

① REVIEW PROBLEM

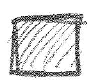
SEE HANDOUT

② THIN-WALLED TUBES

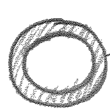
SO FAR, CONSIDERED AXISYMMETRIC SOLID SHAFTS

e.g. 

CAN ATTAIN VERY COMPLEX BEHAVIOR IN NON-AXISYMMETRIC ELEMENTS

e.g. 

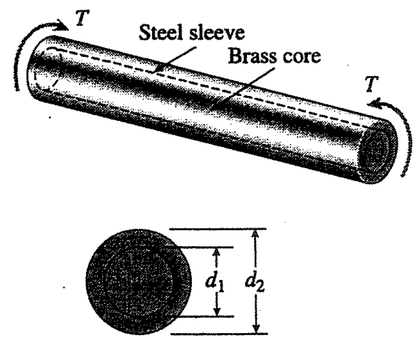
HOW ABOUT THIN-WALLED AXISYMMETRIC SHAFTS?

e.g. 

SINCE THIN WALL, CAN ASSUME SHEAR STRESS is UNIFORMLY DISTRIBUTED.

Problem 3.8-12 The composite shaft shown in the figure is manufactured by shrink-fitting a steel sleeve over a brass core so that the two parts act as a single solid bar in torsion. The outer diameters of the two parts are $d_1 = 40$ mm for the brass core and $d_2 = 50$ mm for the steel sleeve. The shear moduli of elasticity are $G_b = 36$ GPa for the brass and $G_s = 80$ GPa for the steel.

Assuming that the allowable shear stresses in the brass and steel are $\tau_b = 48$ MPa and $\tau_s = 80$ MPa, respectively, determine the maximum permissible torque T_{max} that may be applied to the shaft. (*Hint:* Use Eqs. 3-44a and b to find the torques.)



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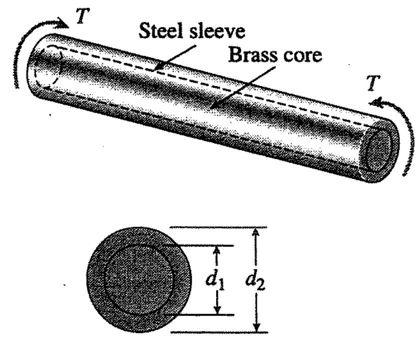
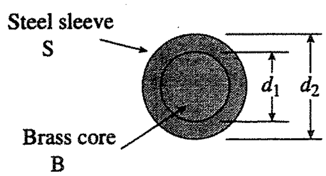


Solution 3.8-12 Composite shaft shrink fit



$d_1 = 40$ mm

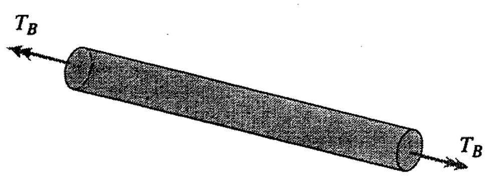
$d_2 = 50$ mm

$G_B = 36$ GPa $\quad G_S = 80$ GPa

Allowable stresses:

$\tau_B = 48$ MPa $\quad \tau_S = 80$ MPa

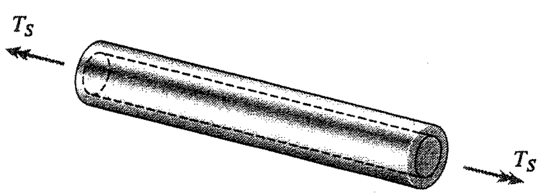
Brass Core (only)



$$I_{PB} = \frac{\pi}{32} d_1^4 = 251.327 \times 10^{-9}\ \text{m}^4$$

$$G_B I_{PB} = 9047.79\ \text{N} \cdot \text{m}^2$$

Steel sleeve (only)



$$I_{PS} = \frac{\pi}{32} (d_2^4 - d_1^4) = 362.265 \times 10^{-9}\ \text{m}^4$$

$$G_S I_{PS} = 28{,}981.2\ \text{N} \cdot \text{m}^2$$

Torques

Total torque: $T = T_B + T_S$

$$\text{Eq. (3-44a): } T_B = T\left(\frac{G_B I_{PB}}{G_B I_{PB} + G_S I_{PS}}\right) = 0.237918\ T$$

$$\text{Eq. (3-44b): } T_S = T\left(\frac{G_S I_{PS}}{G_S I_{PB} + G_S I_{PS}}\right) = 0.762082\ T$$

$T = T_B + T_S$ (CHECK)

Allowable torque T based upon brass core

$$\tau_B = \frac{T_B(d_1/2)}{I_{PB}} \qquad T_B = \frac{2\tau_B I_{PB}}{d_1}$$

Substitute numerical values:

$$T_B = 0.237918\ T = \frac{2(48\ \text{MPa})(251.327 \times 10^{-9}\ \text{m}^4)}{40\ \text{mm}}$$

$$T = 2535\ \text{N} \cdot \text{m}$$

Allowable torque T based upon steel sleeve

$$\tau_S = \frac{T_S(d_2/2)}{I_{PS}} \qquad T_S = \frac{2\tau_S I_{PS}}{d_2}$$

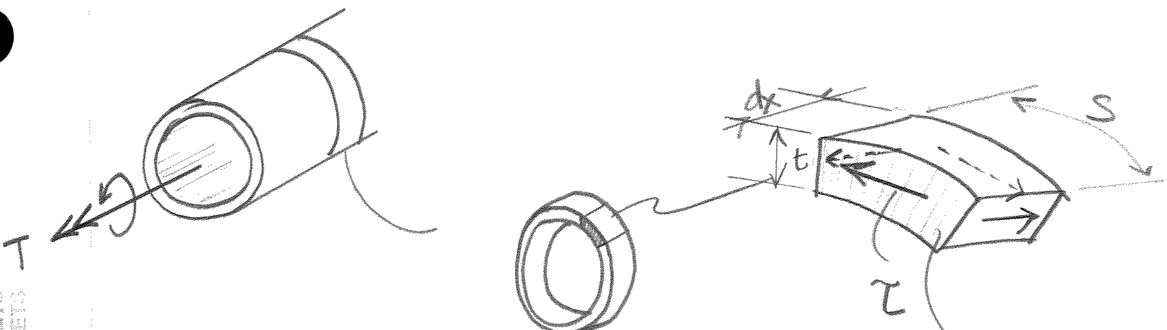
Substitute numerical values:

$$T_S = 0.762082\ T = \frac{2(80\ \text{MPa})(362.265 \times 10^{-9}\ \text{m}^4)}{50\ \text{mm}}$$

$$T = 1521\ \text{N} \cdot \text{m}$$

Steel sleeve governs $\quad T_{max} = 1520\ \text{N} \cdot \text{m}$ ←

CONSIDER TUBE:



ASSUME WALL IS SO THIN, WE HAVE A UNIFORM SHEAR DISTRIBUTION

τ_{AVG}

FOR THIN WALLED MEMBERS, WE CAN FIND SHEAR FLOW

$$\boxed{q = \tau_{AVG}\, t}$$

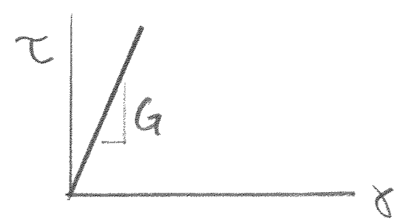
thickness

UNITS $\frac{lb}{in}$, $\frac{N}{m}$, etc.

(CHAPTER 5.8 OUT)

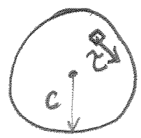
③ INELASTIC TORSION

- SOFAR, ASSUMED MATERIAL IS PERFECTLY LINEAR



- BUT IF WE GO BEYOND A LINEAR REGION, THEN REQUIRE PLASTIC ANALYSIS

- FIRST, CONSIDER EQUILIBRIUM:



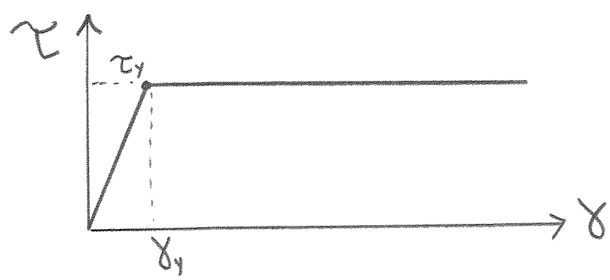
FORCE ON SQUARE IS $\tau \, dA$

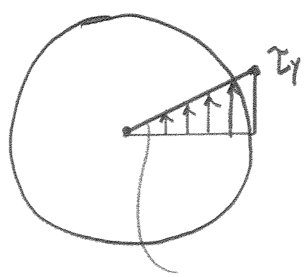
$$T = \int_A \rho \, \tau \, dA \qquad dA = 2\pi\rho\, d\rho$$

$$\boxed{T = 2\pi \int_0^c \tau \rho^2 \, d\rho}$$

function of ρ

- Let US INCREMENTALY LOAD SHAFT
  ASSUME BI-LINEAR MATERIAL



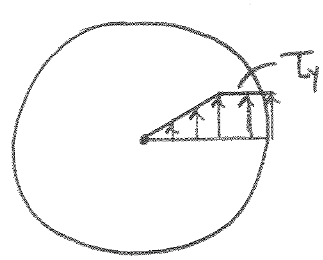


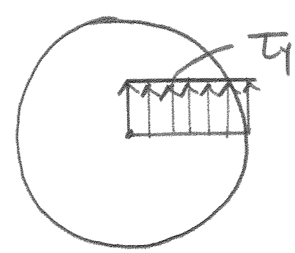
$$\tau = \frac{Tc}{J}$$

$$T = 2\pi \int_0^c \frac{\tau_y}{c} \rho^3 \, d\rho$$

$$= \frac{2\pi \tau_y}{c} \frac{c^4}{4} = \frac{\pi \tau_y c^3}{2}$$

$$\boxed{T_y = \frac{1}{2} \pi \tau_y c^3}$$





$$T_p = 2\pi \int_0^c \tau_y \rho^2 \, d\rho$$

$$\boxed{T_p = \frac{2}{3} \pi \tau_y c^3}$$

$$\boxed{T_p = \frac{4}{3} T_y}$$