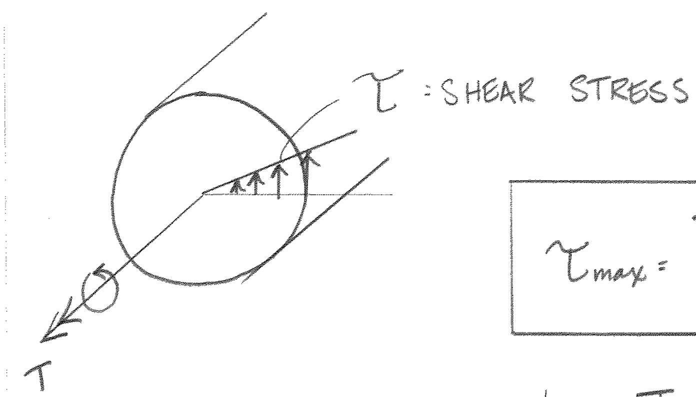
CLASS #12: BEHAVIOR OF RODS & SHAFTS IN TORSION

OBJECTIVES:
1. REVIEW TORSION FORMULA
2. CALCULATION of POLAR MOMENT of INERTIA
3. DEFINE ANGLE of TWIST
4. PRACTICE PROBLEM

1 TORSION FORMULA



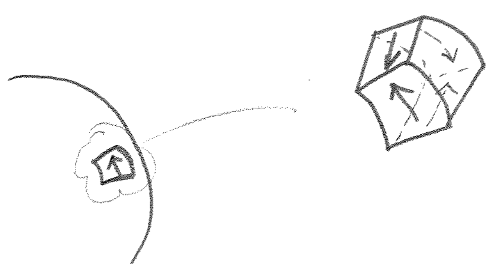
$$\tau_{max} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{J}$$

where J = POLAR MOMENT of INERTIA

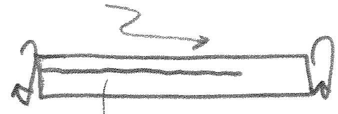
$$= \int_A \rho^2 \, dA$$

CONSIDER AN IFINTESTIMAL SMALL CUBE ON FACE



∴ TO HOLD CUBE IN EQUILIBRIUM MUST DEVELOP ALONG AXIAL PLANES

FOR EXAMPLE: WOOD ROD



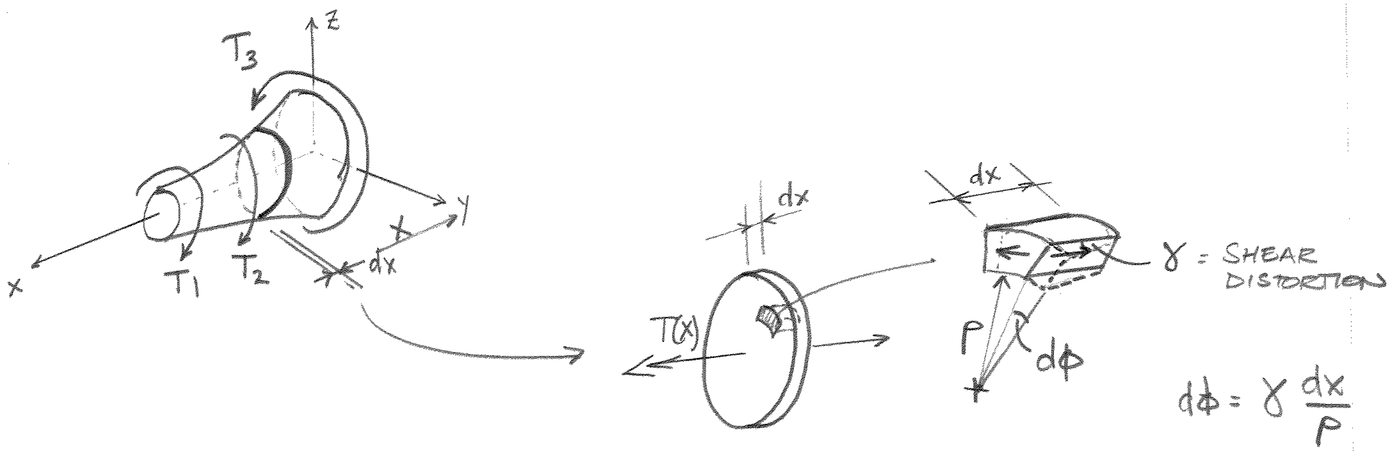
FAILURE IS AXIAL SPLIT. BECAUSE WOOD IS ANISOTROPIC

(SECTION 5.3 - POWER TRANSMISSION IS OUT)

② ANGLE of TWIST

HOW MUCH DOES A SHAFT DEFORM (TWIST) UNER TORQUE?

"ANGLE of TWIST, ϕ"



$$d\phi = \gamma \frac{dx}{\rho}$$

HOOKES LAW: $\tau = G\gamma$

TORSION FORMULA: $\tau = \frac{T(x)\rho}{J(x)}$

$$\therefore \quad \gamma = \frac{T(x)\rho}{GJ(x)}$$

$$d\phi = \frac{T(x)}{J(x)G} dx$$

$$\boxed{\phi = \int_0^L \frac{T(x)}{J(x)G} dx}$$

ϕ IS ANGLE of TWIST BETWEEN TWO ENDS of ROD (RADIANS).

IF CONSTANT TORQUE & CROSS SECTION :

$$\boxed{\phi = \frac{TL}{JG}} \longrightarrow$$ LOOK FAMILIAR?

$$\delta = \frac{PL}{EA}$$

↓

HOW G IS DETERMINED FOR MATERIAL

APPLY TORQUE, T, AND MEASURE TWIST, ϕ

③ STATICALLY INDETERMINATE TORQUE LOADED MEMBERS

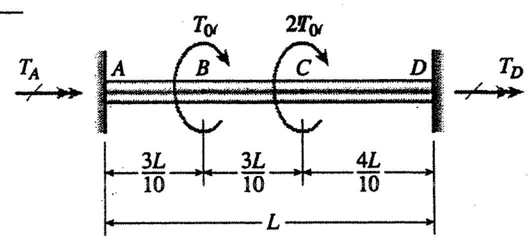
FOR STATICALLY INDETERMINATE TORQUE LOADED SHAFTS, AGAIN APPLY <u>COMPATIBILITY</u>

CONSIDER AN EXAMPLE . . .

Statically Indeterminate Torsional Members

Problem 3.8-1 A solid circular bar *ABCD* with fixed supports is acted upon by torques T_0 and $2T_0$ at the locations shown in the figure.

Obtain a formula for the maximum angle of twist ϕ_{max} of the bar. (*Hint:* Use Eqs. 3-46a and b of Example 3-9 to obtain the reactive torques.)



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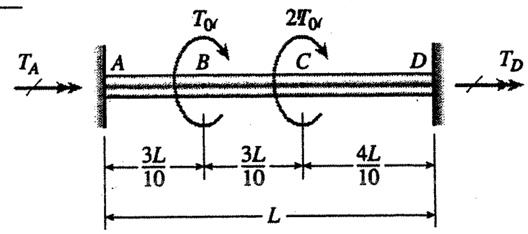
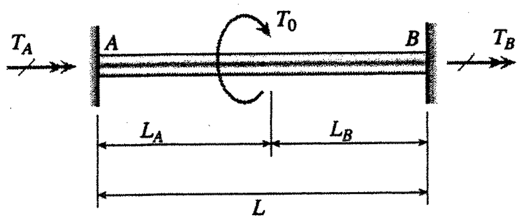


Solution 3.8-1 **Circular bar with fixed ends**



From Eqs. (3-46a and b):

$$T_A = \frac{T_0 L_B}{L}$$

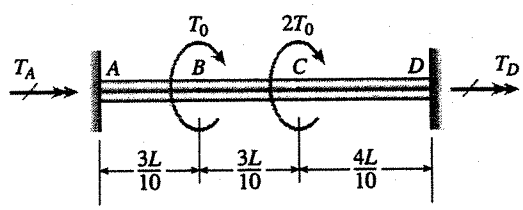
$$T_B = \frac{T_0 L_A}{L}$$

APPLY THE ABOVE FORMULAS TO THE GIVEN BAR:

$$T_A = T_0\left(\frac{7}{10}\right) + 2T_0\left(\frac{4}{10}\right) = \frac{15T_0}{10}$$

$$T_D = T_0\left(\frac{3}{10}\right) + 2T_0\left(\frac{6}{10}\right) = \frac{15T_0}{10}$$

ANGLE OF TWIST AT SECTION *B*



$$\phi_B = \phi_{AB} = \frac{T_A(3L/10)}{GI_P} = \frac{9T_0L}{20GI_P}$$

ANGLE OF TWIST AT SECTION *C*

$$\phi_C = \phi_{CD} = \frac{T_D(4L/10)}{GI_P} = \frac{3T_0L}{5GI_P}$$

MAXIMUM ANGLE OF TWIST

$$\phi_{max} = \phi_C = \frac{3T_0L}{5GI_P} \quad \longleftarrow$$