CLASS 11 - TORSION MEMBERS

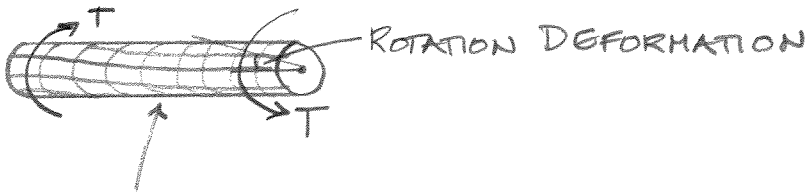
OBJECTIVES: ① DEFINE TORQUE
② DERIVE TORSION FORMULA

① TORQUE:

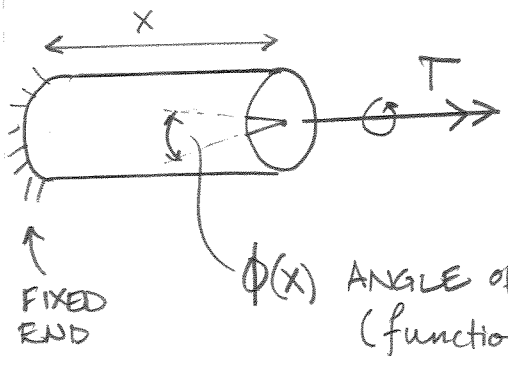
- TORQUE is a MOMENT THAT TWISTS A MEMBER ABOUT A LONGITUDINAL AXIS.

e.g. SHAFTS IN CAR ENGINE

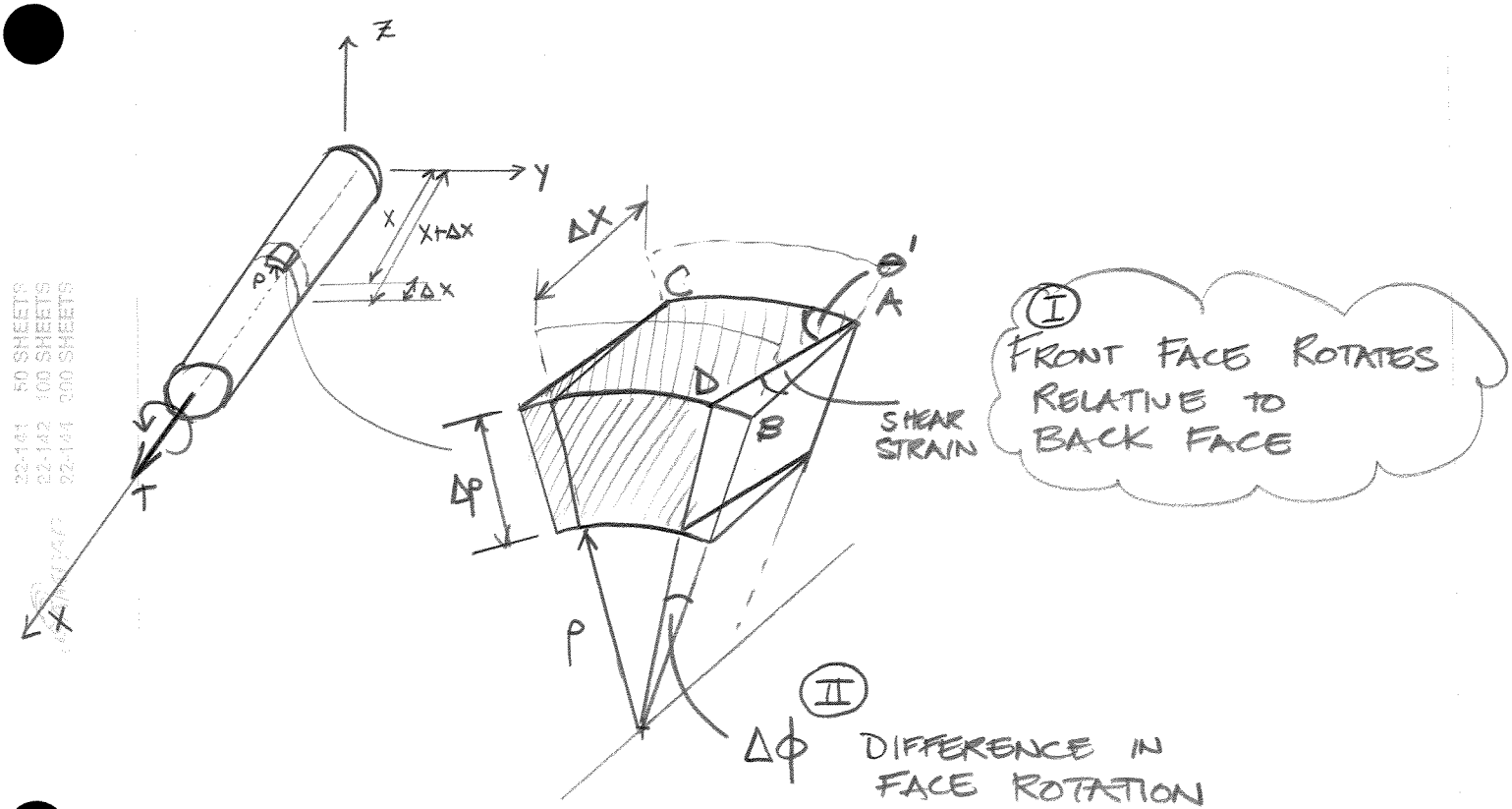
- IF WE TWIST A DEFORMABLE ELEMENT:



- PLANE SECTIONS REMAIN PLANE
- SMALL ANGLE ⟶ - ENDS DO NOT WARP
- RADIUS REMAINS UNCHANGED



② UNDERSTAND DISTORTION – INFINTESIMALLY SMALL ELEMENT



(I) FRONT FACE ROTATES RELATIVE TO BACK FACE

(II)

(III)

– OBSERVE : CLEARLY SHEAR STRAIN

(IV) SHEAR STRAN : $\gamma = \frac{\pi}{2} - \lim_{\substack{C\to A \\ B\to A}} \theta'$

Let $\Delta x \to dx$, $\Delta\phi \to d\phi$

$$BD = \rho\, d\phi = dx\,\gamma$$

$$\therefore \boxed{\gamma = \rho \frac{d\phi}{dx}}$$

WHAT DOES THIS MEAN?

- SHEAR STRAIN VARIES LINEARLY WITH DISTANCE FROM AXIS, ρ.
  ($d\phi/dx$ IS CONSTANT FOR SECTION)

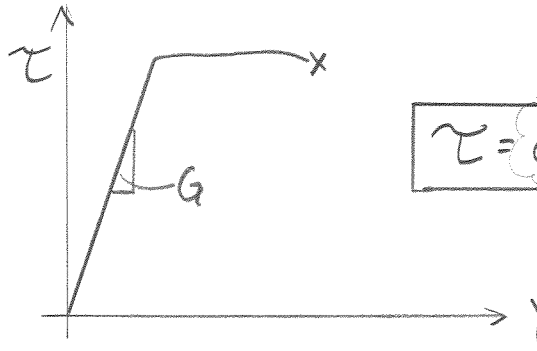
IF "c" IS TUBE RADIUS:

$$\gamma = \left(\frac{\rho}{c}\right)\gamma_{max}$$

OCCURS AT TUBE SURFACE ($\rho = c$)

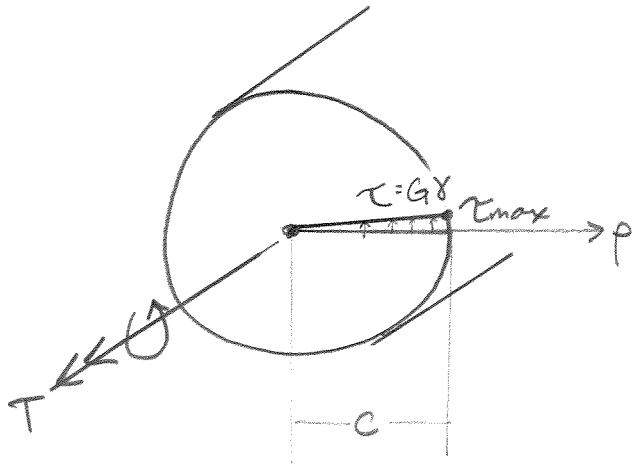
③ TORSION FORMULA

RECALL HOOKE'S LAW IN SHEAR



SHEAR MODULUS

$$\boxed{\tau = G\gamma}$$



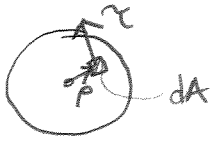
$$\tau = G\left(\rho\,\frac{d\phi}{dx}\right)$$

$$\tau_{max} = G\gamma_{max}$$

$$\therefore\ \tau = \left(\frac{\rho}{c}\right)\tau_{max}$$

ELEMENT MUST BE IN EQUILIBRIUM

$$T = \int_A (\tau\, dA)\,\rho$$



$$T = \int_A \rho \left(\frac{\rho}{c}\right) \tau_{max}\, dA$$

$$= \frac{\tau_{max}}{c} \underbrace{\int \rho^2 dA}_{\text{POLAR MOMENT of INERTIA, } J}$$

$$\therefore \boxed{\tau_{max} = \frac{Tc}{J}}$$

&

$$\boxed{\tau = \frac{T\rho}{J}}$$

TORSION FORMULA