ABSTRACT

A thermomechanical model for a shape memory alloy (SMA) wire under uniaxial loading is implemented in a finite element framework, and its results are compared with new experimental data. The constitutive model is a one-dimensional continuum model of an SMA element, including two internal field variables, strain gradient effects, possible unstable mechanical behavior, and the relevant thermomechanical couplings resulting from latent heat effects. The model is calibrated to recent experiments of typical commercially available polycrystalline NiTi wire. The shape memory effect and pseudoelastic behaviors are demonstrated numerically as a function of applied loading rate and environmental parameters, and the results are found to be quite similar to experimental data. The model is then used to simulate a simple SMA actuator device, and the model proves to be a useful tool to assess the performance.

Keywords: Shape memory alloy, phase transformation, constitutive model, thermomechanical coupling, thermodynamics, localization, finite element analysis, actuator.

1. INTRODUCTION

Shape memory alloys (SMAs) are known for two remarkable properties, the shape memory effect and pseudoelastic behavior. These arise from the well known temperature-induced and/or stress-induced reversible martensitic transformations (see Refs. 1 and 2). The most popular SMA for applications is nearly equiatomic polycrystalline NiTi (usually in wire form) due to its excellent structural properties, corrosion resistance, fatigue resistance, and superior memory capability compared to other SMAs. The number of applications involving NiTi SMAs seems to be growing rapidly in recent years, especially in the biomedical and aerospace fields (see, for example, Refs. 3, 4, and 5). The behavior is rather complex, however, and often leads to difficulties for design and application. The material exhibits complicating factors, such as an extreme thermomechanical coupling, unstable mechanical behavior, and localized self-heating/self–cooling due to latent heat effects. These factors lead to propagating transformation fronts during stress–induced transformation and extreme rate and environment sensitivities. In this work, we develop and implement a relatively comprehensive uniaxial constitutive model of a commercially available polycrystalline NiTi wire and investigate these aforementioned sensitivities. The constitutive model is a simplified version of the one developed in Ref. 6. In Section 2 the thermomechanical boundary value problem is developed, including the coupled equilibrium and heat equations and the kinetic law. Numerical and experimental results are presented in Section 3, showing the fundamental isothermal responses, the shape memory effect, and non–isothermal pseudoelastic responses. Section 4 presents a simulation of a simple uniaxial SMA actuator to demonstrate the usefulness of the model as a potential design and analysis tool.

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2. THERMOMECHANICAL PROBLEM AND CONSTITUTIVE MODEL

2.1. Boundary value problem

The thermomechanical boundary value problem of interest consists of a thin polycrystalline SMA wire of cross-sectional area, $A$, and length, $L$ (stress free austenite). Rigid grips are attached at each end (reference coordinates $x = 0$ and $x = L$), and the wire is immersed in a fluid medium. The grips can provide mechanical loads by an end-displacement, $\delta(t)$, applied at $x = L$, and they are assumed to act as perfect heat sinks at a user-prescribed temperature $\theta_g(t)$. The ambient fluid medium provides no mechanical loads, but acts as a temperature bath held at a user-prescribed temperature $\theta_a(t)$. The temperature of the grips and the fluid bath can be controlled separately. An electrical voltage, $V_e(t)$, may also be applied to the wire, creating an internal heat source in the form of Joule heating.

The framework for the coupled thermomechanical problem follows the approach described in Ref. 6. The state of the wire is determined by the following field variables: the displacement field $u(x, t)$, the absolute temperature $\theta(x, t)$, and two internal variables $\xi_1(x, t)$ and $\xi_2(x, t)$ that represent the mass fractions of tensile and compressive martensite, respectively. The mass fraction of austenite is implicitly defined by mass conservation as $1 - \xi_1 - \xi_2$. The fact that each of these mass fractions must be non-negative defines a triangular admissible space for $\xi_1$ and $\xi_2$ as shown in Fig. 1a. In this way, pure austenite ($A$) is defined by $(\xi_1, \xi_2) = (0, 0)$, tensile martensite ($M^+$) is $(1, 0)$, compressive martensite ($M^-$) is $(0, 1)$, and twinned martensite ($M^+/M^-$) is $(1/2, 1/2)$. Pseudoelastic and shape memory effect paths are shown in Fig. 1b and Fig. 1c, respectively.

2.2. SMA constitutive model

Constitutive relations for the SMA wire are based on a specific Helmholtz free energy $\phi(\varepsilon, \varepsilon', \theta, \xi)$, where $\varepsilon$ is the nominal strain, $\varepsilon' = \partial \varepsilon / \partial x$ is the strain gradient, and $\xi$ is the phase fraction vector ($\xi_1, \xi_2$). As discussed in Ref. 6, the dependence on the strain gradient is necessary to regularize the deformation field during unstable mechanical behavior. The constitutive model described below is similar to the one in Ref. 6, except that the chosen form of the chemical part of the free energy has been simplified. The free energy has three parts,

$$\phi(\varepsilon, \varepsilon', \theta, \xi) = \phi_E(\varepsilon, \varepsilon', \xi) + \phi_C(\theta, \xi) + \phi_T(\theta),$$

where $\phi_E$ is the elastic energy, $\phi_C$ is the chemical energy, and $\phi_T$ is the phase independent thermal energy. The elastic strain energy density is chosen as

$$\rho \phi_E = \frac{E_A + (\xi_1 + \xi_2) \Delta E}{2} [\varepsilon - (\xi_1 - \xi_2) \beta]^2 + \frac{\gamma}{2} \varepsilon'^2,$$

where $\rho$ is the mass density, $E_A$ is the modulus of pure austenite, $\Delta E = E_M - E_A$ is the difference in the effective martensite and austenite moduli (a negative quantity), $\beta$ is the stress-free transformation strain (a material constant), and $\gamma$ is the strength of the strain gradient effect (a constant, depending on the initial
aspect ratio of the wire). Thermal expansion is negligible compared to the transformation strain. The form of the specific chemical free energy is chosen as

$$\phi_C = - (\theta - \theta_R) (\xi_1 + \xi_2) \Delta s,$$

where $\theta_R$ is a reference transformation temperature and $\Delta s = s_M - s_A$ is the entropy change from austenite to martensite (a negative quantity) which is related to the latent heat of transformation. Lastly, the remaining, purely thermal, part of the free energy is

$$\phi_T = (c_0 - s_0) (\theta - \theta_R) - c_0 \theta \log(\theta/\theta_R),$$

where $s_0$ and $c_0$ are material constants representing the phase-independent specific entropy and specific heat, respectively.

Following Coleman and Noll thermodynamics (see Refs. 7 and 8) and the strain gradient theory of Mindlin, the non-negativity requirement of the local entropy production leads to the following Gibb’s relations

$$s = - \phi, \theta,$$

$$\dot{\sigma} = \rho \phi, \varepsilon,$$

$$\ddot{\sigma} = \rho \phi, \varepsilon',$$

where $\dot{\sigma}$ and $\ddot{\sigma}$ are the first and second order nominal stresses, respectively. Entropy production arises, therefore, only from the phase transformation and heat flux terms, and so we require

$$\mu : \dot{\xi} \geq 0,$$

$$-q_x \theta' \geq 0,$$

where the chemical driving force vector (relative to austenite) is defined as $\mu \equiv - \phi \xi$. The inequality (6) involving the heat flux ($q_x$) is satisfied if we assume the conventional axial heat conduction law

$$q_x = -K \theta',$$

where $K$ is a positive thermal conductivity for the material. This is assumed to follow the linear mixture rule,

$$K(\xi) = K_A + (\xi_1 + \xi_2) \Delta K,$$

where $K_A$ is the thermal conductivity of pure austenite, and $\Delta K = K_M - K_A$ is the difference in conductivity between martensite and austenite. It is known, for example, that the thermal conductivity may change by a factor of two depending on the phase (see Ref. 10).

2.3. Governing equations

The coupled thermomechanical problem has three times scales: the inertial time scale, the inherent martensitic transformation time scale, and the thermal time scale. The time scale for inertial effects based on the elastic wave speed of the material is roughly an order of magnitude smaller than the time scale for phase transformation. The time scale for either axial heat conduction or lateral heat convection is much larger than either of these. Consequently, the governing field equations are mechanical equilibrium (ignoring inertial effects), the heat equation, and a relatively stiff kinetic relation for phase transformation.

2.3.1. Equilibrium

Mechanical equilibrium within $[0, L]$ is $F' = 0$, where $F$ is the axial force and

$$F = \dot{\sigma} A - (\ddot{\sigma} A)'',$$

The mechanical boundary conditions are

$$\dot{T} A = \dot{\sigma} A - (\ddot{\sigma} A)'', \quad \ddot{T} = \ddot{\sigma}.$$

where $\dot{T}$ and $\ddot{T}$ are the first and second order referential end tractions, respectively. At the ends ($x = 0, L$) the higher order traction, $\ddot{T}$, is set to zero, thereby providing the necessary higher order boundary condition.
2.3.2. Heat equation

The heat equation within \([0, L]\) is

\[
\rho A \dot{Q}_s = (KA\theta')' - HP(\theta - \theta_a) + \frac{V_e^2}{R_e L},
\]

(11)

where \(\dot{Q}_s\) is the specific heat storage rate defined as \(\dot{Q}_s = \theta \dot{s} - \mu \cdot \xi\). The first term on the right hand side of Eq. (11) represents the axial heat flux per unit length, the second term represents the lateral heat flux per unit length with convective film coefficient \(H\) and wire circumference \(P = 2\sqrt{\pi}A\), and the third term is the distributed Joule heat source with wire electrical resistance \(R_e\). For the chosen form of the free energy the specific heat storage rate can be written as

\[
\dot{Q}_s = c_0 \dot{\theta} - (\mu - \theta \mu_{\theta}) \cdot \xi.
\]

(12)

The two terms on the right hand side of Eq. (12) are the rate of sensible heat change and latent heat change, respectively. The thermal boundary conditions are simply \(\theta(0, t) = \theta(L, t) = \theta_g(t)\), indicating the heat sink nature of the grips.

2.3.3. Kinetic law

The kinetic relation determining \(\dot{\xi}\) must satisfy the entropy condition (6). We choose the following kinetic law

\[
\dot{\xi} = \begin{cases} 
0 & \text{if } \mu \cdot \mathbf{m} \leq \mu_c, \\
v_0(\mu \cdot \mathbf{m} - \mu_c)\mathbf{m} & \text{if } \mu \cdot \mathbf{m} > \mu_c,
\end{cases}
\]

(13)

where \(\mathbf{m}\) is the unit vector direction of phase transformation and \(v_0\) is the slope of the kinetic law. The parameter \(\mu_c\) is the critical chemical potential necessary for phase transformation to proceed. It produces the hysteretic nature of the martensitic transformation.

The direction of phase transformation is assumed to be colinear with the chemical driving force, except at the boundary of the admissible phase region (see again Fig. 1). If \(\xi_i > 0\) for all \(i = 1, 2, 3\) (recall the third fraction, austenite, is \(\xi_3 = 1 - \xi_1 - \xi_2\)) the phase fraction coordinates reside in the interior of the admissible space and \(\mathbf{m} = \mu/||\mu||\), where \(||\mu|| = \sqrt{\mu \cdot \mu}\). If exactly one \(\xi_i = 0\) (for \(i = 1, 2, 3\)) the coordinates reside on an edge of the space and the direction is chosen to point inward or colinear with the edge, as

\[
\mathbf{m} = \begin{cases} 
\mu/||\mu|| & \text{if } \mu \cdot \mathbf{n}^{(i)} < 0, \\
t^{(i)} & \text{if } \mu \cdot \mathbf{n}^{(i)} \geq 0,
\end{cases}
\]

(14)

where edge \(i\) of the admissible space has unit tangent vector \(t^{(i)}\) and unit outward normal vector \(\mathbf{n}^{(i)}\). If the coordinates reside in a corner where \(\xi_i = 0\) and \(\xi_j = 0\) (for \(i, j = 1, 2, 3\) and \(i \neq j\)) then the direction is chosen according to

\[
\mathbf{m} = \begin{cases} 
\mu/||\mu|| & \text{if } \mu \cdot \mathbf{n}^{(i)} < 0 \text{ and } \mu \cdot \mathbf{n}^{(j)} < 0, \\
t^{(k)} & \text{if } \mu \cdot \mathbf{n}^{(k)} \geq 0 \text{ and } \mu \cdot t^{(k)} > 0 \text{ for } k = i \text{ or } j, \\
0 & \text{otherwise.}
\end{cases}
\]

(15)

In this case the direction of the two tangents are chosen to enclose the admissible space by an acute angle, such that \(t^{(k)} \cdot \mathbf{n}^{(k)} = 0\) for \(k = 1, 2\) and \(t^{(i)} \cdot \mathbf{n}^{(j)} \leq 0\) and \(t^{(j)} \cdot \mathbf{n}^{(i)} \leq 0\) for \(i \neq j\).

2.4. Calibration of Parameters

The SMA model is calibrated to a typical commercially available, polycrystalline NiTi wire. Pseudoelastic NiTi wire with a 0.76 mm diameter was purchased from Memry Corporation (GUIDE-BB30). Differential Scanning Calorimetry (DSC) shows the various transition temperatures and latent heat values (Fig. 2a). The stress–free phase at room temperature is austenite, therefore the wire is pseudoelastic at room temperature. Note that the DSC curve also shows the presence of the rhombohedral (R) phase during cooling, which is not modeled.
in the aforementioned constitutive relations, since it plays a relatively minor role. Several thermomechanical experiments were performed on wires with a free length of approximately 60 mm between the grips. Figure 2b shows displacement–controlled mechanical experiments each at an end–displacement rate of $\dot{\delta}/L = 5 \times 10^{-5} \text{s}^{-1}$ at a different ambient air temperature. These were used to fit the material parameters shown in Table 1 for the SMA constitutive model. Note that a permanent inelastic strain remains after unloading for the experiments conducted at 40$^\circ$C and above, an effect that is not yet included in the constitutive model. In addition, the wires used at the lowest four temperatures (-50$^\circ$C to -20$^\circ$C) were first quenched in liquid nitrogen so that the initial phase is mostly twinned martensite rather than R–phase.

**Table 1.** Model parameters and physical constants.

<table>
<thead>
<tr>
<th>Known</th>
<th>Fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ = 60 mm</td>
<td>$\beta$ = 0.0474</td>
</tr>
<tr>
<td>$A$ = 0.456 mm$^2$</td>
<td>$E_A$ = 74.88 GPa</td>
</tr>
<tr>
<td>$\rho$ = 6.5 x 10$^6$ g/m$^3$</td>
<td>$E_M$ = 20.46 GPa</td>
</tr>
<tr>
<td>$R_e$ = 0.132 $\Omega$</td>
<td>$\gamma$ = 2000 N</td>
</tr>
<tr>
<td>$K_A$ = 28 W/mK</td>
<td>$H$ = 40 to 4000 W/m$^2$K</td>
</tr>
<tr>
<td>$K_M$ = 14 W/mK</td>
<td>$\theta_R$ = 251 K</td>
</tr>
<tr>
<td>$c_0$ = 0.5 J/gK</td>
<td>$\sigma_0$ = 1 J/gK</td>
</tr>
<tr>
<td>$\rho \theta_R \Delta s$ = -15.2 J/g</td>
<td>$\mu_c$ = 1.077 J/g</td>
</tr>
<tr>
<td>$v_0$ = 2 g/J s</td>
<td></td>
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</table>

Figure 3a shows the predicted end–displacement controlled mechanical responses from the constitutive model for several ambient temperatures, each assuming uniform deformation and isothermal conditions (obtained numerically with a single finite element with artificially large values of $\gamma$ and $H$). Note that the local (homogeneous) responses at pseudoelastic temperatures have unstable transformation paths. These arise naturally as a result of the decrease in elastic modulus from austenite to martensite. (Interestingly, SMA models which neglect this modulus change predict local homogeneous stress–induced transformation along a flat trans-
Figure 3. SMA constitutive model: (a) homogeneous, isothermal stress–strain behavior, and (b) inhomogeneous, isothermal force–displacement responses (solid line is numerical, dotted line is experimental).

formation path). In any event, this softening transformation path leads to localization and propagation in the strain and temperature fields once the uniform deformation assumption is removed, as will be discussed in the next section. At the lowest temperatures (see -50°C) the initial state is twinned martensite ($M^+/M^-$) and the end–displacement induces detwinning until pure $M^+$ is created. This process mimics an elastic–perfectly plastic (frictional) material.

### 3. EXPERIMENTAL AND NUMERICAL RESULTS

The SMA constitutive model and thermomechanical boundary value problem of Section 2 is implemented in an in–house finite element research code called SARPP$^{11}$. The length of the SMA wire is discretized by finite elements, each of which has Hermite cubic shape functions for the displacement field (two displacement degrees of freedom (DOFs) per node) and linear shape functions for the temperature field (one temperature DOF per node). Weak forms of the equilibrium equation and the heat equation are developed in terms of these DOFs, and the solution procedure is marched forward in time in a staggered manner. For each time increment the equilibrium equation is first solved by Newton–Raphson iteration holding the temperature DOFs fixed, and then the heat equation is solved holding the displacement DOFs fixed, and so on. The kinetic law is incorporated by an explicit 4th–order Runge–Kutta algorithm that has been linearized to produce a consistent tangent between the strain increment and stress increment for the equilibrium iteration and an estimate for the update in the internal variables. In this way, the first and second order stresses and the internal variables are updated at the gaussian quadrature points of each element.

#### 3.1. Isothermal mechanical responses

The isothermal mechanical responses are now rerun with 100 finite elements (see Fig. 3b) under end–displacement control, $\delta(t)$, ramped up and then down at a slow displacement rate ($\dot{\delta}/L = 5 \times 10^{-5}s^{-1}$). The cross-sectional area distribution at the ends of the wire are reduced by 15% locally to simulate the stress concentrations that exist at the grips in the experiments of Fig. 2b. This imperfection effectively eliminates any nucleation peak seen in the pseudoelastic responses during the $A \rightarrow M^+$ transformation during loading ($\dot{\delta} > 0$), since the transformation nucleates at the imperfections. However, small nucleation dips are seen in some of the calculated
responses at the beginning of the reverse transformation $M^+ \rightarrow A$ during unloading ($\dot{\delta} < 0$), which is also often seen in the experiments, since reverse nucleation occurs somewhere in the central region of homogeneous deformation.

### 3.2. Shape memory effect

Figure 4a shows an experiment where an SMA wire was first quenched in liquid nitrogen and then allowed to warm up nearly stress–free in an environmental chamber (air) to a low temperature (-50 °C) and then stretched–unloaded in displacement control, leaving a 4% residual strain. Under load–control at a very small tensile force (to avoid buckling of the wire) the temperature is then increased to room temperature, and the strain is recovered between -10 °C and 0 °C. An isothermal displacement–controlled pseudoelastic loop is then performed on the wire (just for reference). Figure 4b shows the corresponding simulation of the shape memory effect and pseudoelastic response.

![Figure 4](image)

**Figure 4.** Shape memory and pseudoelastic sequence: (a) experiment, and (b) simulation.

### 3.3. Pseudoelastic responses

We now look at the pseudoelastic responses in detail. Figure 5 (dashed line) shows an experiment taken from Iadicola and Shaw\(^1\)\(^2\) where thermoelectric devices were used to adjust the temperature profile along the length of the wire to force initial nucleation events to occur somewhere in the midspan of the length (away from the grips). During loading, the grip temperature was held above that of the center region, and during unloading, the grip temperature was held below that of the center region. One can readily see a rather large nucleation peak for the onset of the $A \rightarrow M^+$ transformation and then an upside down nucleation peak at the onset of the $M^+ \rightarrow A$ transformation. Both of these features are captured in the simulation of Fig. 5 (solid line) where no imperfection is included. Further transformation occurs at the Maxwell stress (or propagation stress) for that transformation.

The nucleation peaks in the mechanical response represent elastic energy barriers that must be surmounted for additional nucleations to occur. The mechanism for this to occur comes from local self–heating (or self–cooling) due to the release (or absorption) of latent heat that raises (or lowers) the local propagation stress. Since equilibrium requires the force to be constant along the length, a non–uniform temperature field can result in additional nucleation events. Therefore, isothermal conditions tend to produce only one nucleation during a transformation, but more adiabatic conditions tend to produce numerous nucleations. See Refs. 13, 14, and 15 for more detailed discussions of this issue. Figure 6a shows four experiments in room temperature air at increasing displacement rates, showing both the mechanical responses and the evolution of temperature obtained...
from infrared imaging. The specimen is tagged at two locations with targets used by a laser extensometer for strain measurement, so the streaks seen in the infrared images are caused by stray reflections off these targets and should be disregarded. Figure 6b shows the corresponding simulations with a film coefficient $H = 40 \text{ W/m}^2\text{K}$ chosen to represent stagnant air. Each simulation includes a global mechanical response and a contour plot of the evolving temperature field. The trends seen in the experiments are captured in the simulations. Specifically, at low rates there are few fronts (one or two), thus the self–heating/self–cooling is relatively minor and the response resembles an isothermal one. However, the cases at the higher displacement rates result in more severe self–heating/self–cooling, more numerous fronts, and the pseudoelastic responses become distorted (seemingly stabilized). The agreement shown is reasonable, but could probably be improved somewhat with more precise knowledge of the ambient film coefficient $H$.

4. A DESIGN CASE STUDY

As a final demonstration, we now consider a prototype uniaxial actuator. It consists of an SMA element in parallel with a prestrained bias spring. Figure 7a shows the assembly sequence and then operation of the actuator. In state (1) the actuator is unassembled with a bias spring (spring constant $k_B = 0.05 E_A A/L$) that is initially longer than the stress–free SMA element by an amount $\delta_B (= 0.0948 L)$. The actuator is assembled in the cold state (state (2)), and sufficient prestrain and prestress creates detwinned martensite in the SMA. In state (2) an external spring is then attached that represents the stiffness (spring constant $k_E = 0.025 E_A A/L$) of some external structure that the actuator will do work against. When the SMA is heated, it transforms back to austenite and the actuator contracts (state (3)). When the SMA is cooled again the actuator resets (state (2)) due to the bias spring, and the actuator can be cycled between these two states by alternate heating/cooling.

The simulated performance of this actuator immersed in -50°C air is shown in Fig. 7b. The bottom plot shows the history of the applied electrical voltage which is used to heat the SMA element. The next two plots upward show the history of the temperature and the phase fraction, respectively, at the midspan of the SMA. The second plot from the top shows the normalized forces in the SMA ($F_S$), the bias spring ($F_B$), and the external spring ($F_E$). The top plot shows the normalized displacement of the left end of the bias spring ($\delta_B$) and the right end of the SMA actuator ($\delta_S$). Initially at time $t = 0$ the SMA is stress–free twinned martensite, $\xi = (0.5, 0.5)$, at -50°C. During the first 10 seconds the voltage is off and the actuator is assembled by prestraining the bias spring ($\delta_B$), thereby causing stretching of the SMA ($\delta_S$) and detwinning to $\xi = (1, 0)$.

For 50 seconds starting at time $t = 10$ s the voltage is ramped up to 0.274 V and held. At time $t = 60$ s the voltage is ramped down to zero and held for 50 seconds. This cycle is then repeated during the next 100 seconds. The voltage change is calculated to give a steady state temperature change using the approximation $\Delta \theta_{\infty} = V_e^2 / (HPR \rho L) = 100\degree C$. While the voltage is applied, one can see the temperature rise of the midpoint of the SMA wire. The temperature rate is governed initially by the sensible heat change roughly according to the time constant $\tau_\theta = \rho Ac / (HP) = 15.5$ s, coming from the approximate formula for the temperature rise $\Delta \theta = \Delta \theta_{\infty} \left( 1 - \exp^{-t/\tau_\theta} \right)$ where conduction and phase transformation are ignored. However, this is interrupted
\[ \frac{\delta}{L} = 1 \times 10^{-5} \text{s}^{-1} \quad \frac{\delta}{L} = 1 \times 10^{-4} \text{s}^{-1} \quad \frac{\delta}{L} = 1 \times 10^{-3} \text{s}^{-1} \quad \frac{\delta}{L} = 1 \times 10^{-2} \text{s}^{-1} \]

\[ \Delta \theta (\circ C) \]

\[ \delta \frac{\delta}{L} / (\% \text{LE}) \text{(a)} \]

\[ \delta \frac{\delta}{L} / (\% \text{LE}) \text{(b)} \]

**Figure 6.** Pseudoelastic responses at different displacement rates in room temperature air: (a) experiments, and (b) simulations.

At time \( t = 29.1 \text{ s} \) when the SMA begins to transform to austenite and latent heat must be absorbed. Once the transformation is complete near \( t = 49.8 \text{ s} \) the temperature rate accelerates again. A similar evolution of temperature is seen when the voltage is turned off at \( t = 60 \text{ s} \). A rapid cooling occurs until \( t = 71.4 \text{ s} \) when the SMA transforms back to detwinned martensite and latent heat is released, thereby arresting the temperature drop momentarily. The simulation is useful, therefore, in determining how much time is necessary for the sensible heat change versus latent heat change (in this case 30.3 s and 19.7 s, respectively), both of which contribute to the effective time response of the actuator. During heating, one can also see the tensile force in the SMA increase above its prestress value, the SMA wire contract (decrease in \( \delta S \) from 0.0548 to 0.0366), the bias spring compression increase in magnitude, and the external spring tensile force increase. The product of the force in the external spring and the change in SMA displacement from its prestrained value is related to the mechanical work the actuator performs on the external environment. For this simulation, the calculated specific external work, i.e., the work performed on the external spring per unit mass of SMA, is approximately \( W_E = k_E \Delta \delta^2 / (2 \rho AL) = 0.048 \text{ J/g} \).

Figure 8a shows the evolution of temperature profiles with time during the two heating/cooling cycles.
Figure 7. Prototype 1-D SMA actuator: (a) schematic of assembly and operation, (b) simulation results: evolution of displacements, forces, midspan phase fraction, midspan temperature, and applied voltage.

Figure 8. SMA actuator simulation: (a) evolution of temperature profiles, and (b) evolution of $M^+$ phase fraction.

temperature field is not uniform, since the ends are in contact with thermal heat sinks at the low ambient temperature. In addition, Fig. 8b shows the evolution of the distribution of $M^+$ during these thermal cycles. Interestingly, only the middle third, or so, of the SMA wire participates in the transformation, due to the lower temperatures imposed at the ends.

No attempt has been made so far to optimize the performance of the actuator. It will be interesting
to investigate in the future which combinations of bias and external springs and ambient environment would improve the performance. In addition, the temperature and phase fraction distributions shown in Fig. 8 suggest that insulating the ends, rather than holding the temperature of the ends fixed, might be an effective strategy to increase the fraction of SMA length participating in the transformation, thereby increasing the specific work performed.

5. SUMMARY AND CONCLUSIONS

A constitutive model for a shape memory alloy (SMA) wire and the corresponding thermomechanical boundary value problem for uniaxial loading was implemented in a finite element framework. The one–dimensional continuum model of an SMA included two internal field variables (phase fractions), strain gradient effects, possible unstable mechanical behavior, and the relevant therm–mechanical couplings resulting from latent heat effects. The model was calibrated to recent experiments of typical commercially available polycrystalline NiTi wire. The isothermal force–displacement responses and the shape memory effect were shown both numerically and experimentally. The pseudoelastic response was investigated as a function of applied loading rate, showing trends similar to experiments. Specifically, as the situation became more adiabatic (at relatively high loading rate and in a relatively insulating environment) more self–heating and self–cooling could occur resulting in turn in more nucleation events and more transformation fronts and distorted force–displacement responses. Finally, the performance of a simple prototype actuator was simulated, showing the regions of active transformation and predicting the rate of actuation. The model and simulation scheme should prove to be a useful tool in the future for studying the design of such actuators and their performance.

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