INITIATION AND PROPAGATION OF LOCALIZED DEFORMATION IN ELASTO-PLASTIC STRIPS UNDER UNIAXIAL TENSION

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Abstract—This paper deals with the evolution of inhomogeneous deformation in shape memory alloy strips and mild steel strips under uniaxial tension. New experiments on NiTi strips, which initially are in an austenitic phase, show that at a critical stress level martensite nucleates in sharp bands inclined at 55° to the axis of loading. Under prescribed end displacement martensite subsequently spreads either by steady-state propagation of inclined transition fronts or via a criss-cross pattern of finger-like features. Similar events have been reported in the literature regarding the evolution of Lüders bands in fine grained steel strips and wires. The similarity of macroscopic events, despite the different mechanisms of instability at the micro-level, prompted us to approximate the material behavior as a finitely deforming elasto-plastic solid with a trilinear up-down-up nominal stress-strain response. Two such stress-strain responses were used in finite element simulations of strip tension tests. In the first the true stress-strain response maintains its stability and in the second the intermediate branch has a negative slope. While both material models produced inhomogeneous deformations with features similar to those of the experiments, the larger initiation peak associated with the second gave results which closely resembled specific experiments. The numerical simulations confirmed that the evolution of events seen in experiments on SMAs and mild steels is strongly influenced by overall geometric (structural) effects. Furthermore, the success of this simple continuum constitutive model strongly suggests that continuum level events remain dominant players in such fine grained materials. © 1998 Elsevier Science Ltd. All rights reserved

I. INTRODUCTION

Shape memory alloys (SMAs), such as NiTi, exhibit reversible, stress-induced transformations between two solid states called austenite (A) and martensite (M). It has been demonstrated (Shaw and Kyriakides, 1995) that in some temperature regimes \( A = M \) phase transformations result in inhomogeneous deformations. For example, in a displacement controlled, uniaxial, isothermal test such transformations result in well defined stress plateaus (one for loading and one for unloading) during which phase boundaries propagate along the length of the specimen. The material processing of certain NiTi SMAs leaves a thin brittle oxide layer on the surface of the alloy. In recent experiments on thin strips of nearly equiatomic NiTi, the macroscopic deformation associated with the

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transformation shattered this oxide layer resulting in reflectivity changes and contrast in
the color of the transformed and untransformed portions of the specimen. This enabled
full field monitoring of the evolution of such inhomogeneous deformations (Shaw and
Kyriakides, 1997).

In this paper results from new experiments of this type are presented to illustrate the
generic details of the transformation fronts. Several similarities are identified between
the events observed in the present experiments and events reported in the literature from
uniaxial experiments on strips of mild steel, which experience inhomogeneous deforma-
tions due to Lüders bands. Motivated by these macroscopic similarities a simple model is
presented which captures the main experimental features.

II. EXPERIMENTAL OBSERVATIONS

II.1. Uniaxial experiments on NiTi strips

An example of a uniaxial experiment on a NiTi strip is shown in the photographic
sequence in Fig. 1. The photographs were taken at 20 s intervals during the loading phase
of a 50.8 mm long (L) strip of this material. The strip had a uniform width (a) of 4.0 mm
and was 0.40 mm thick (same batch of material as that in Shaw and Kyriakides, 1997). It
was pulled at ambient temperature at a constant end displacement rate (\(\delta\)) of 5.08 \(\mu\)m s\(^{-1}\).
At this temperature the unstressed material is in the \(A\) phase which corresponds to black
in the photographs. Fig. 2(a) shows the corresponding nominal stress history. Initially, A
deforms elastically and the specimen deforms homogeneously (see 1 in Fig. 1). Martensite
(light gray in photographs) nucleates at \(t_1\) at the lower end, due to the inherent stress
concentration at the grip, at a nominal stress of 403 MPa. The stress immediately drops by
10 MPa and an angled front separating the two phases starts to propagate from the bot-
tom as seen in configurations 2–4 in Fig. 1. The position of the front as a function of time
was measured from each photograph and is plotted as solid dots in the \(x - t\) diagram in
Fig. 2(b). As pointed out in the preceding publications, the \(A \rightarrow M\) transformation is

![Fig. 1. Photographs of NiTi strips (at 20 s intervals) during stress-induced transformation from \(A\) (black) to \(M\)
(white) (\(\delta/L = 10^{-4}\) s\(^{-1}\)).](image)
exothermic. As a result, even at this relatively slow displacement rate, the specimen experiences self heating in the neighbourhood of the propagating transformation front. The stress required to transform the material is strongly influenced by temperature (Fig. 7 in Shaw and Kyriakides, 1997). Thus, the local increase in temperature causes the increase in transformation stress seen to occur between \( t_1 \) and \( t_2 \) (stress increased by \( \sim 28 \text{ MPa} \)). At \( t_2 \) the stress exceeds the stress required to nucleate \( M \) at the upper grip. Subsequently, two fronts propagate from the grips towards the center of the specimen (5–29 in Fig. 1).

We demonstrated in the past (Shaw and Kyriakides, 1995, 1997) that if the specimen is pulled at a relatively slow rate, coexisting fronts propagate at the same velocity \( c \) given by

\[
c = \frac{\dot{\delta}}{n \Delta \varepsilon_t}
\]  

(1)

where \( n \) is the number of currently propagating fronts and \( \Delta \varepsilon_t \) is the transformation strain. The construction lines in Fig. 2(b) are based on eqn (1). The top line is in perfect
agreement with the data while the bottom lines deviate somewhat. (We suspect that this is due to some slippage of the clamped part of the specimen at the lower grip.)

At $t_3$ the two fronts coalesce and subsequent deformation is again homogeneous. Loading is terminated at $t_4$. Despite the relatively slow rate of the test, the environment could not dissipate heat at the rate that heat was being released by the transforming material. Consequently, the temperature of the propagating fronts increased gradually (by 4°C close to $t_3$), and thus, the stress increased by approximately 40 MPa during this time interval.

The specimen is left to return to ambient temperature and is then unloaded at the same displacement rate. When the stress drops to a low enough level $M$ transforms back to $A$ and deformation is fully recovered (so called pseudoelastic behavior). The events associated with reverse transformation are not relevant to issues of interest here, and so are they are not included in this write-up (e.g. Fig. 2 and discussion in Shaw and Kyriakides, 1997).

Several such experiments were conducted (not shown here) where the rate of loading and the specimen geometry (width, length and shape) were varied. At displacement rates higher by one or two decades than $\delta = 5.08 \mu m \ s^{-1}$, the number of nucleation sites and emanating fronts was larger than in the experiment in Fig. 1. For slower rates transformation spread via just one propagating front. By and large, the velocity of the fronts obeyed eqn (1) irrespective of their number.

Using specially designed specimens it was possible to show that the nucleation stress of $A$ is distinctly higher than the stress required to propagate the transformation (see Fig. 7 and related discussion in Shaw and Kyriakides, 1997). By contrast, the stress required to nucleate $M$ during unloading is lower than that required to continue the transformation. Thus, each of the transformations exhibits similar macromechanical characteristics to those of other problems which exhibit propagating instabilities (Kyriakides, 1993).

Using full field temperature profiles of the test specimens obtained by an infrared imaging radiometer, together with synchronous photographic sequences of the evolution of transformation, it was possible to relate the events observed in these experiments to thermomechanical interactions between the latent heats of transformation, the nucleation stresses of each phase and their respective sensitivity to temperature changes. More on each of these issues along with quantitative results can be found in Shaw and Kyriakides, (1997).

The details of the transformation fronts are interesting and worthy of further attention. Some of the observed features are summarized below:

- In longer specimens ($L \sim 10 a$ or longer) and when two or more fronts coexisted in the test section, the fronts were usually sharp angled discontinuities as shown in the photograph in Fig. 3(b) (at least according to the visualization technique we used). The angle between the front and the direction of loading ranged between 51° to 60°, but occasionally angles as high as 67° were measured. At the higher angles the fronts exhibited some curvature as shown in Fig. 3(c).
- In several experiments the evolution of transformation was monitored by a low magnification microscope and a more detailed record of events was captured on video (at 30 fr s⁻¹). From these records it was possible to extract pictures of the nucleation of the new phase (Fig. 4(a)). Within the accuracy of the measurements ($\sim \pm 1°$) the initial band angle was 55° in all cases.
Fig. 3. (a) Definition of geometric variables of inclined band. (b) Photograph of two diverging $A \rightarrow M$ transformation fronts. (c) Photograph of a single moving $A \rightarrow M$ transformation front with some curvature.

Fig. 4. (a) Photograph of nucleation of $M$ in $A$ (inclination of band $\alpha = 55^\circ$). (b) Multiple propagating fronts during $M$ (black) to $A$ (grey) transformation (unloading). (c) Criss-cross propagation fronts during $A$ (black) to $M$ (light grey) transformation (loading).
In order to accommodate the steep gradient in deformation across such fronts, the specimen developed kinks in the plane of the strip (Fig. 3). The kink angle (γ in Fig. 3(a)) was found to depend on the axial position of the front, on the specimen geometry, and on the axial stress. Kink angles ranging from 0.5–2° were measured. Such kinking resulted in bending of the specimen and was more pronounced during the $M \rightarrow A$ transformation due to the lower axial stress. A photograph of a bent strip taken during unloading is shown in Fig. 4(b). This specimen was pulled at a displacement rate of $\bar{\dot{s}} = 50.8 \, \mu m \, s^{-1}$, that is, 10 times faster than the rate in Fig. 1. As a result, it developed multiple nucleation sites and 10 coexisting fronts.

During loading, the grips are preferred nucleation sites in a uniform width strip due to the stress concentrations. In some cases, the orientations of the two fronts which emanated from the grips were in incompatible directions as for example in configurations 5–9 in Fig. 1. This combination of front orientations causes excessive misalignment and bending of the strip. In such cases, one of the fronts tended to switch from an inclination of $-\theta$ to $+\theta$ via a temporary criss-cross pattern (see configurations 9–11 in Fig. 1). The new arrangement of fronts seen in configurations after 11, reduces the overall misalignment of the specimen and minimizes bending. As a result, this arrangement of two parallel fronts remained unaltered until the transformation was completed.

An indication that a switch in front orientation was imminent was the development of some curvature in the front profile such as the one shown in Fig. 3(c). The first finger-like structure initiated at the place of highest curvature (the “nose” of the front).

The coalescence of two fronts propagating from opposite directions (e.g. at $\tau_2$ in Fig. 1) resulted in a detectable stress valley during loading (or peak during unloading) (e.g. see Fig. 11 in Shaw and Kyriakides (1995) and Fig. 3 in Shaw and Kyriakides (1997)). We believe that such stress valleys are due to the interaction of stress-concentrations associated with the neighbourhood of each front. This interaction results in lowering the net stress required to transform the last sliver of material between the two fronts.

For relatively short specimens, like the one with aspect ratio of approximately 3.5 shown in Fig. 5, the front propagated in a different mode. In the case shown first nucleation of $M$ occurred at the upper grip in the form of an inclined band. Initially, a straight inclined front propagated downward, but about one third of the way down it switched to the alternating criss-cross pattern of wedge-like “fingers” which persisted for the remainder of the transformation. A byproduct of this behavior is small islands of untransformed material left in the wake of the propagating front which will only transform at a higher stress. The average angles of these fingers were also in the range of 50–60°. This pattern allows the specimen axis to remain relatively straight and free of kinking. The stress history recorded during criss-crossing exhibited small undulations which were absent during steady-state propagation of clean inclined fronts. The criss-cross pattern was occasionally observed in uniaxial tests on longer strips as well, as the one shown in Fig. 4(c).

II.2. Evolution of Lüders bands in tension tests on mild steel strips

As these experimental observations accumulated, it became increasingly clear that the macromechanical similarities between stress-induced transformations in a SMA and the
development of Lüders bands in a mild steel specimen tested uniaxially are not by chance. Figure 6 shows a stress-elongation response of a typical fine grained, low carbon steel obtained under similar conditions to the ones described above for NiTi. The initial elastic response is terminated at the upper yield stress ($\sigma_U$) when local plastic deformation, known as Lüders strain (bands for strips), begins in the specimen. Subsequently, the Lüders deformation spreads through the specimen very much like in Fig. 1 while the stress remains constant at the lower yield stress ($\sigma_L$). When the whole test section has been deformed to the Lüders strain ($\Delta\varepsilon_L$), the material hardens and the specimen deforms homogeneously once more. The values $\sigma_U$, $\sigma_L$ and $\Delta\varepsilon_L$ depend on the grain size (the finer the grain the higher the values of $\sigma_U$, $\sigma_L$ and $\Delta\varepsilon_L$) and are sensitive to temperature and to the rate of loading.

![Graph showing force-elongation response of mild steel exhibiting Lüders strain.](image-url)
The phenomenon has been widely studied and a rich literature exists on almost all of its aspects. The volume by Hall (1970) presents a comprehensive review of the subject and includes an extensive list of relevant literature. Of particular interest to us were several pioneering works in which the evolution of the Lüders strain in wire and strip specimens was monitored. Hall (1951, 1952) deduced the evolution of Lüders fronts by employing an optical reflection method to record on photographic paper the motion of the centers of wire specimens; Lomer (1952) used metallographic etching and an opaque-stop microscope to examine the interface between deformed and undeformed regions in detail; Butler (1962) coated strip specimens with a brittle lacquer (stress-coat) which cracked at some value of strain resulting in reflectivity changes that enabled full field monitoring of the evolution of Lüders deformation (very similar to our results on SMA strips reported here and in Shaw and Kyriakides, 1997). The following results were reported:

- Sharp, simple bands such as the one shown in Fig. 7(c) formed in strip and wire specimens when the grain size was small (approximately 20 µm and smaller). In specimens with grains of a few µm the band edge was as sharp as one or two grains. Conversely, relatively thick specimens developed complex band patterns and large grain specimens developed diffuse bands.
- Lomer (1952) measured strip band angles of 48–65° and Butler (1962) reported angles of 50–70°.
- All three observed that when the bands were sharp specimen kinking of the order of 1–2° occurred, although no correlation of the kink angle to the Lüders strain was attempted.
- Butler (1962) measured front velocities. He noted that in specimens with uniform grain size the velocity of fronts obeyed eqn (1) (developed independently by Shaw and Kyriakides, 1995 for SMAs).
- In uniform mild steel strip specimens, which were clamped at the ends, fronts often emanated from stress concentrations at the grips and propagated towards the center of the specimen as shown in the photograph in Fig. 7(b). The similarities of this photograph with the one in Fig. 7(a) from our experiments is quite striking.
- Butler's stress histories exhibited small stress valleys when two converging fronts coalesced at the end of the stress plateaus (see Butler, 1962 Fig. 2, and Fig. 6 here).

In summary, despite the differences in the micromechanisms behind the stress-induced transformation in NiTi and the dislocation governed phenomenon of Lüders strain, the macromechanical events in strip specimens have too many similarities for them to be coincidental. This indicated to us that in fine grained NiTi (grain size on the order of a few microns) and fine grained mild steel strips, macroscopic geometric effects and the underlying mechanism of continuum strain localization must play an important role in the phenomena observed. Although there were those who thought the front orientation in mild steel was related to the average of the slip systems of individual grains (Jaoul, 1961), others concluded, as we have, that these features are driven by macroscopic effects (Lomer, 1952; Butler, 1962). Interestingly, this issue does not seem to have been settled for Lüders phenomena (for a recent review of the subject see Estrin and Kubin, 1995).
Motivated by the above conclusions, we will represent the mechanical behavior of the material via the simplest continuum model which will allow us to capture the main macro- mechanical features observed in our NiTi strips. Several of the NiTi strip experiments are simulated by large scale finite element calculations. The objective of these calculations is to establish how effective a continuum-type model can be at reproducing the morphology of deformations observed in experiments.

The literature on strain localization, that is, on the events associated with loss of stability of the material, is quite extensive and a review will not be attempted here. Instability at the material level in many problems results in loss of uniqueness, which in finite element calculations, is exhibited as sensitivity of the solution to mesh size (e.g. Ortiz et al., 1987). A distinct difference between such localization studies and the present problem(s) is that local deformation is eventually arrested here when the material stiffness recovers at the completion of the $A \rightarrow M$ transformation (or at the exhaustion of the Lüders strain). It appears that even when the material model adopted is unstable for a certain deformation
regime, the recovery of material stability at larger strains has an overall stabilizing effect on the solution.

III.1. Constitutive model

In this first attempt at modelling this type of inhomogeneous deformation, we intentionally put aside factors like rate of loading, thermal interaction of the specimen with the environment, and sensitivity of the transformation stress to temperature, all of which can affect the stress-deformation response of NiTi strips. Furthermore, the reversible nature of such deformations will be forgotten since we will be concerned with just loading. (For a theoretical framework with a complete representation of the uniaxial thermomechanical behavior of SMAs, see Abeyaratne and Knowles, 1993). We will consider strictly isothermal, rate independent behavior and assume the material to behave as a finitely deforming $J_2$-type elastoplastic solid which hardens isotropically. A crucial feature of the constitutive model is a trilinear stress-strain response described below.

The room temperature uniaxial true stress ($\sigma$)-logarithmic strain ($\varepsilon$) response of our NiTi SMA is represented as shown in Fig. 8(a) (identified as material I). The tangent modulus of the middle segment was chosen to have a small positive slope. This representation translates into a nearly trilinear force-elongation response with the first and third branches closely resembling the measured responses of uniformly deforming $A$ and $M$ phases, respectively, while the nearly flat middle branch in Fig. 8(a) translates into a monotonically decreasing nominal stress in Fig. 8(b). This "softening" behavior in the force-displacement response is necessary to obtain the desired strain localization. On the other hand, limiting the amount of softening, such that the true stress-logarithmic strain behavior is always rising, is purposely imposed to avoid the well known mesh sensitivity in problems where the material becomes unstable. Consequently, the difference between the

![Graphs](https://example.com/fig8.png)

Fig. 8. Material I: Uniaxial stress-strain response used in finite element analyses. (a) True stress-logarithmic strain, (b) engineering stress-strain.
nucleation stress and the plateau stress of the actual NiTi response in Fig. 8(b) must be limited. This restriction may cause some differences between the predicted and observed events. Nevertheless, it will be demonstrated that even with this idealization many of the macromechanical features observed in SMA and mild steel strips can be reproduced.

The closest example with similar material characteristics is high density polyethylene, which can develop a propagating neck during a uniaxial tension test (e.g. G’Sell and Jonas, 1979; G’Sell et al., 1983; Tugcu and Neale, 1990; Wu and Van der Giessen, 1995). The force–elongation response and the true stress-logarithmic strain response used by Hutchinson and Neale (1983) to analyze this problem (and by a number of other publications on the subject that followed) have similar characteristics to those of our idealized material (see Fig. 1 in Hutchinson and Neale, 1983).

III.2. Strip geometry

Several three-dimensional strip geometries were analyzed using the model discussed above in order to investigate the effects of length, boundary conditions and geometric imperfections on the inhomogeneous deformation features. In all cases the width of the strip \( a \) was 10 times the thickness \( t = 0.40 \text{ mm} \), corresponding to the cross sectional dimensions of the NiTi strips used in the experiments. Strip lengths in the range of \( 3a \) and \( 12.67a \) were considered. The strips were loaded axially under displacement control by prescribing appropriate boundary conditions at the ends. All other surfaces were stress free. Three representative strip configurations (Fig. 9) have the following characteristics:

![Side Imperfection](image)

![Geometric details of strips used in analyses](image)

Fig. 9. Geometric details of strips used in analyses.
III.2.1. Case A. Figure 9(a) shows the geometry of our base case about which parameters will be varied. It is a relatively short strip \((L = 3 \, \text{a})\) with the following boundary conditions

\[
  u(0, \, y, \, z) = 0, \quad v(0, \, 0, \, z) = 0 \quad \text{and} \quad u(L, \, y, \, z) = \delta
\]

where \(\delta\) is prescribed incrementally. These boundary conditions allow the strip to contract freely as a result of the Poisson effect, and the top end is free to displace in the \(y\)-direction. In this and all other case the mid-thickness was assumed to be a plane of symmetry. This allows only half of the strip to be modeled.

III.2.2. Case B. In this case (Fig. 9(b)) the strip has the same geometry as Case A, but the top end is restrained from lateral motion in the \(x-y\) plane by requiring that

\[
  v(L, \, 0, \, z) = 0
\]

in addition to the boundary conditions (2).

III.2.3. Case C. This configuration is used to simulate experiments in longer NiTi strips \((L = 12.67 \, \text{a})\). The deformation patterns of interest were antisymmetric about the mid-span of the specimen (assumed to be at \(x = 0\)). Thus, by imposing the following conditions at mid-span it was possible to limit the analysis to only half of the strip length.

\[
  u(0, \, 0, \, z) = 0, \quad v(0, \, 0, \, z) = 0, \\
  u(0, \, y, \, z) = -u(0, \, -y, \, z), \quad v(0, \, y, \, z) = -v(0, \, -y, \, z) \quad \text{and} \quad w(0, \, y, \, z) = w(0, \, -y, \, z).
\]

The boundary conditions at the upper end \((x = L/2)\) were similar to Case B, namely

\[
  u\left(\frac{L}{2}, \, y, \, z\right) = \frac{\delta}{2} \quad \text{and} \quad v\left(\frac{L}{2}, \, y, \, z\right) = 0.
\]

In all cases the instability was initiated at a chosen location by introducing a geometric imperfection in the form of a small indentation along one side of the strip (denoted in the FE mesh plots as a sideways \(\sqrt{v}\)). The indentation was one thickness wide, 0.02 \(t\) deep and had the shape shown in Fig. 9 (symmetric about its middle, with an exponentially decaying amplitude). This type of imperfection allowed the localization to develop without any artificial directional bias across the width. For example, a thickness imperfection across the width (Tvergaard et al., 1981) also worked nicely, but it required one to choose its inclination.

III.3. Discretization

The strips were discretized with three-dimensional, 20-node (quadratic), isoparametric, reduced integration, brick elements using ABAQUS*. The elements had one-to-one aspect ratio in the \(x-y\) plane to obtain as “isotropic” a mesh as possible in order to avoid

*We are grateful to Hibbitt, Karlsson and Sorensen, Inc. for making ABAQUS available under academic license.
introducing any preferred directions into the results. The mid-plane \( z = 0 \) was assumed to be a plane of symmetry and the half thickness was represented by a single element. The density of the mesh in the \( x-y \) plane was determined by convergence studies, such that all relevant features of the strain fields were represented without becoming computationally prohibitive (Shaw, 1997). The width was usually discretized by twenty elements (Fig. 9(a) and (b)). In the longer specimens (Case C) the main interest is in the deformation patterns that develop close to the ends of the specimens, so for computational expediency, two mesh densities were used as shown in Fig. 9(c). The coarser mesh in the inner part of the strip had only one half the number of elements across the width as the finer mesh used in the outer parts. Nodes with common coordinates at the interface of the two meshes were tied together. The displacements of other interface nodes (in the finer mesh) were interpolated to ensure compatible deformations across the interface.

IV. RESULTS AND DISCUSSION

IV.1. Initiation of instability

In Section II.1. it was noted that the nucleation of \( M \) in our uniaxial tests on NiTi strips was found to consistently occur in narrow zones inclined at 55° to the direction of loading (Fig. 4(a)). It is well known that this is also the inclination of necks which tend to form in uniaxial tension tests in thin metal strips (e.g. Nadai, 1950, pp. 319–320). Hill (1952) treated the onset of such a neck as a discontinuity in deformation. At the critical stress the material inside the narrow band is allowed to deform with a velocity \( \dot{u} \) discontinuity oriented at an angle \( \Psi \) from the band while the material on either side of it remains in a state of uniaxial stretching (Fig. 10). As a result of the constraint provided by these adjoining, relatively undeformed, sectors of the strip, the strain along the band (in direction of \( \dot{t} \)) can be assumed to be zero. Under these conditions \( \Psi \) is related to the principal strain rates inside the band (\( \dot{\varepsilon}_1 \) and \( \dot{\varepsilon}_2 \)) through

\[
\sin \Psi = \frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_2}{\dot{\varepsilon}_1 - \dot{\varepsilon}_2}.
\] (6)

Since for this problem

\[
\dot{\varepsilon}_1 = -2\dot{\varepsilon}_2
\] (7)

it follows that

\[
\Psi = \sin^{-1} \left( \frac{1}{3} \right) = 19.47^\circ \quad \text{and} \quad \alpha = \frac{\pi}{4} + \frac{\Psi}{2} = 54.74^\circ.
\] (8)

The band could be inclined at \( \pm \alpha \) to the \( x_1 \)-axis; the directions represented by the vector \( \dot{t} \) and the corresponding one along the \( -\alpha \) direction are characteristics preferred by such discontinuous local deformations. (These results were first derived by Bijlaard, 1940 and then by Hill, 1948 for a biaxially loaded thin sheet which could yield anisotropically.)

Although the underlying cause for the onset of localized deformation in NiTi is different from that of structural metals, the macroscopic constraints on the deformation inside
the band are applicable to both families of materials. The plastic deformation in these calculations is assumed to be incompressible. The transformations in NiTi are essentially volume preserving so the results also apply to this material. The applicability of these results to our NiTi alloy is also supported by the experimental results presented, and as we will see below, by the results of the numerical simulations.

IV.2. Results from strip Case A

The results of the numerical simulation of a uniaxial tension test on a strip with the characteristics of Case A in Section III.2. are shown in Figs 11–16. The strip had a small side imperfection at the location of \( \psi \) in the mesh in Fig. 12. The calculated overall force-elongation response is shown in Fig. 11(a). The nominal stress first rises sharply to a value of 402 MPa while the deformation of the strip is homogeneous. Localized deformation is initiated at the imperfection and the average stress quickly drops down to the Maxwell value (Ericksen, 1975). The stress remains relatively unchanged as in the experiments while the deformation spreads through the specimen. When the whole specimen has been deformed to a strain corresponding to \( \Delta \varepsilon_t \) ("transformation" strain), the stress plateau is terminated, the response becomes stiff again, and subsequent deformation is homogeneous once more. An expanded view of the stress plateau is shown in Fig. 11(b) and a detailed view of the neighbourhood of the load maximum is shown in Fig. 11(c).

Two sequences of deformed configurations of the strip are shown in Fig. 12, corresponding to the numbered flags on the response in Fig. 11. The first sequence of seven configurations shows in detail the onset of localized deformation in the neighbourhood of the initial load maximum. The second sequence of 10 configurations shows the subsequent evolution of deformation in the strip specimen. We have chosen to show color contours of
Fig. 11. (a) Force–displacement response for Case A (material 1), (b) expanded view about stress plateau, (c) expanded view about initiation peak.

$\varepsilon_x$ since this is the most representative measure of the extent of the deformation (is very close to the maximum principal strain to which cracking of the oxide layer on the NiTi strips is thought to be most sensitive). A color legend is included for each sequence. The first sequence spans logarithmic strains from 0.5 to 1% while the second spans 0 to 5%.

The part of the $P - \delta$ response in the neighbourhood of the initial load maximum is expanded in Fig. 11(c) for clarity. Configuration 1 in Fig. 12, just before the load
maximum, has a small concentration of strain around the local imperfection. At the load maximum two narrow fingers of deformation emanate from this area and "quickly" spread across the strip as shown in configurations 2 and 3. The two zones of localized deformation are oriented at ± 55° to the axis of loading. While the overall load is dropping, deformation localizes in the upper band (see configurations 5–7 and then 1 and 2 in the second sequence). Because of the coarser strain range used in the second sequence, the second band is not visible, but it does remain in the background as an alternative mechanism for localized deformation.

The deformation in the band has a significant shear component which results in kinking of the strip. This kinking is illustrated in Fig. 13 which shows two plots of the transverse displacement, \( v(x) \), of the axis of the strip as a function of the net axial displacement \( \delta \). The displacement profiles that correspond to the 10 configurations of the second sequence in Fig. 12 are identified with flags in Fig. 13. Figure 13(a) shows that as the deformation in the band grows the kinking of the specimen increases and reaches a maximum value just after configuration 2. Although the upper edge of the strip is free to translate in the y-direction, the extent of kinking is counteracted by the axial load which tends to align the strip. In fact, following a local minimum the axial force is seen in Fig. 11(b) to increase slightly due to specimen misalignment. The increase in stress is sufficient to activate the secondary characteristic in the upper and lower boundaries of the band. These new zones of localized deformation are visible in configuration 3. They have the effect of reducing the misalignment of the specimen and cause a corresponding reduction in stress (Fig. 11(b)). The specimen now has two kinks, resulting in the formation of a ridge in the

![Fig. 13. Lateral displacement of strip axis of Case A (material I) as a function of \( \delta \). (a) Forward view, (b) backward view.](image-url)
Fig. 12. Sequence of axial strain contours for Case A (material I).

Fig. 18. Sequence of axial strain contours for Case B (material I).
Fig. 28. Sequence of axial strain contours for Case D (material II).
displacement profiles in Fig. 13 and less transverse displacement at the upper end of the strip.

The more symmetric pattern which results from these events is seen in a transitory stage in configuration 4. In configurations 5–7 the symmetric pattern consisting of two V-shaped propagating fronts is fully developed, the propagation of deformation reaches steady-state, and the load level has stabilized near the Maxwell load (Fig. 11(b)). As more of the strip deforms, the overall kinking of the axis of the strip is reduced. This is reflected in the gradual lowering of the height of the ridge in the displacement profiles in Fig. 13. Interestingly, the edges of the initial band remain inclined at 55° to the axis of loading during broadening (see configuration 2). Furthermore, the edges of the V-shaped fronts, in the steady-state part of the simulation, are also inclined at ±55°. The front profiles are seen to develop some curvature during the transition from the two parallel fronts to the V-shaped ones.

Because of the location of the initial imperfection, the front reaches the lower grip first. The final sliver of material is transformed from both directions (just after configuration 8). This causes a temporary relaxation of the overall load level (Fig. 11(b)). The upper front then returns to steady-steady propagation (configuration 9) and the load level briefly returns to the Maxwell level. The inhomogeneous deformation in the strip is completed.
just after configuration 10 with another relaxation in load associated with the deformation of a final sliver of material. Subsequent deformation is homogeneous and the load level rises quickly.

As stated before, the front features observed in our NiTi experiments were the result of the disturbance of a brittle oxide layer, giving a black and white picture of regions encompassing the two phases. It was clear that the critical strain level for the oxide layer occurred somewhere along the stress plateau between about 1 and 4.5% strain. In order to obtain a more direct comparison between the calculated and the experimental results, a black and white version of the axial strain contours was post-processed from the finite element calculations. The exact strain level at which the oxide layer shattered, however, is unknown. We felt that, although the essence of the observed events was not affected by the strain level at which the oxide shattered, some of the finer features of the fronts may have been affected. To assess the effect of this variable we varied the level of the cutoff strain used to produce the black-white contours. Figure 14 shows the effect this has on the deformation pattern of configuration 4 of Fig. 12. In this case, the features changed from a V-shape front for cutoff strain less than 2% (~25% of Δεr) to the straighter banded fronts for cutoff strains greater than 2%. The features most representative of those seen in the NiTi strip experiments are observed for the 2% cutoff strain where the fronts are angled bands but with emanating finger-like patterns. Therefore, this 2% strain cutoff will be used to produce black and the white axial strain contours that follow.

The black-and-white version of the second color sequence in Fig. 12 produced with a 2% cutoff strain is shown in Fig. 15. The configurations shown correspond to the color ones presented in Fig. 12. The events in the black-and-white configurations are simpler
but very similar to those in color. Deformation starts in one single band with nearly straight fronts. The nature of kinking is shown in the schematic in Fig. 16(a) and illustrated in the expanded view of configuration 2 in Fig. 16(b). Here, the maximum kink angle $\gamma$ is approximately $1.5^\circ$. The band subsequently broadens which results in an increase in the misalignment of the strip axis. In configuration 3 (Fig. 15) the band fronts have developed some curvature which is a prelude to activation of the secondary characteristics. Fingers of deformation along these characteristics are seen to develop in configuration 4. Subsequently, the fronts are V-shaped and the deformation pattern becomes nearly symmetric about the strip axis, resulting in a gradual straightening of the strip as discussed above.

The following observations can be made from these results:

- The geometric characteristics of the initial inclined band, such as its $55^\circ$ inclination, its straight fronts, and the extent of kinking of the strip are in agreement with experimental observations made both in our recent NiTi strip experiments and in low carbon steel strips in the past.
- The curvature that develops in the fronts during the transition from the inclined to the V-shaped configurations and the associated finger-like deformation patterns are similar to what we observed in the NiTi strip experiments (compare Fig. 3(c) and configuration 3 in Fig. 15).
- When converging fronts coalesced the overall force in the specimen exhibited small dips similar to those observed in experiments on both classes of materials.
- The mesh of this calculation has 1200 elements. The calculation was repeated using meshes with 2700 and 4800 elements. This mesh refinement did not affect either the width of the transition zones or the deformation patterns presented in Fig. 12 (Shaw, 1997).
- Clearly, overall structural effects resulting from the end constraints and the axis misalignment due to kinking of the strip, play an important role in the observed events. The end constraints and the net axial tensile force acting on the strip act to minimize specimen misalignment.

In order to illustrate this last point we consider a strip of the same geometry and material but now prevent transverse motion of the top edge by fixing its midpoint (Case B).

IV.3. Results from strip Case B

Results for this case, plotted in a similar fashion to the results of the previous strip, appear in Figs 17–19. The initial geometric imperfection is located at the same place as in the previous case. Because of the additional end constraint, kinking associated with a single inclined band would require a significant increase in the net axial force for it to develop in such a short strip. Instead, this strip simultaneously develops two deformation bands inclined at $\pm 55^\circ$ to the axis of loading as shown in configurations 2–7 of the upper sequence in Fig. 18 and configuration 1 of the lower sequence. The opposing angles of kinking in the two bands result in local misalignment of the axis of the strip which can be seen in Fig. 19, but the two undeformed parts of the specimen remain nearly straight.

The two bands initially broaden (see configuration 2) but the increase in local kinking of the strip axis cannot be sustained. Thus, the deformation switches to the symmetric
V-shaped fronts seen in the previous case. This pattern which is more compatible with the end constraints, is fully developed soon after configuration 3 and the net axial force drops near the Maxwell level. It can be observed in Fig. 19 that with the formation of the two symmetric fronts the misalignment of the axis of the strip is significantly reduced and stays small for the remainder of the deformation history. Subsequent events are qualitatively similar to what was observed in the previous case.

IV.4. Results from strip Case C

We now consider results from one of the longer strips analyzed \((L = 12.67 a)\). Here, the initiation of instability is assumed to occur with the simultaneous appearance of two parallel bands near the upper and lower ends. This produces a deformation pattern which is antisymmetric about the mid-span of the strip. This is preferred as the band broadens in order to minimize specimen misalignment (as for example was the case in the NiTi strip in Fig. 1 after configuration 10). This antisymmetric condition allowed us to limit attention to only half of the strip using the boundary conditions given in eqns (4) and (5).

The results are summarized in Figs 20–22. Figure 20 shows the calculated force-elongation response, and Fig. 21 shows a sequence of 12 deformed configurations. The initial geometric imperfection is located one width away from the top end (see the FE mesh in Fig. 21). The instability initiates from the imperfection (see configuration 1) and is accompanied by a load drop. The deforming zone develops into an inclined band (configuration 2) which resembles those seen in the experiments. This results in a kink in the axis of the strip seen in the plot of \(v(x)\) versus \(\delta\) in Fig. 22. Because of the kinking the initial average axial stress remains somewhat higher than the Maxwell value. The deformation first spreads towards the ends of the strip. The evolution of deformation in these regions is similar to those of the short specimens discussed above. The second local force minimum in Fig. 20 corresponds to the coalescence of converging deformation fronts just after configuration 3. Subsequently, deformation propagates towards the strip mid-span. The inclined fronts can no longer be sustained because of the excessive strip kinking, thus, the fronts gradually revert to the kinematically preferable V-shape. By configuration 6 this new front shape is fully developed, and the specimen axis is seen in Fig. 22 to straighten. From here on the front propagates in a steady-state fashion near the Maxwell load. The analysis was terminated when the front reached the coarser mesh.

Thus, the calculated deformation patterns in the early stages of this exercise resemble patterns seen in experiments on NiTi strips of similar geometry. However, in the simulation the inclined bands can not be sustained for too long; whereas, they remained until the whole specimen was transformed in experiments such as the one shown in Fig. 1. For this material model, strip kinking associated with the inclined fronts requires a somewhat higher net axial stress than the stress required to switch an inclined band to a V-shaped one. This is a limitation due to the compromise we accepted when we adopted the special stress-strain response in Fig. 8(b) in order to maintain the stability of the material. The difference between the modeled initiation stress and the propagation stress was significantly less than that of our NiTi at 25°C (Fig. 8(b)). This larger nucleation peak makes switching of deformation patterns more difficult in the actual material than in the assumed material and, we suspect, is responsible for the differences between the calculated and observed deformation patterns.
Fig. 17. (a) Force-displacement response for Case B (material I), (b) expanded view about stress plateau, (c) expanded view about initiation peak.

We note that mild steels usually exhibit an even larger difference between the upper and lower yield stresses than for our NiTi alloy. This difference is again responsible for the two parallel inclined propagating Lüders fronts being the preferred pattern throughout uni-axial tests on mild steel strips (e.g. Hall, 1951, 1952; Butler, 1962).

To further illustrate the importance of the size of the initiation peak on the results, we will now conduct similar calculations using a true stress-logarithmic strain response with a slightly negative slope as shown in Fig. 23 (designated as material II). By doing this it becomes possible to match the size of the nucleation peak measured in the NiTi at 25°C.
As a result, the solution now exhibits some sensitivity to the FE mesh. Unlike other problems of localization caused by instability of the material (as opposed to the structure, e.g. Ortiz et al., 1987) the presence of the third stable branch in the stress–strain response (after a strain of \(~5\%\)) has an overall stabilizing effect on the solution since it limits the extent of unstable behavior. As a result, the mesh sensitivity observed affected mainly the shape of the transition zones between the relatively undeformed and the deformed regions of the strip (Shaw, 1997). In order to facilitate comparison of the results obtained from FE calculations using materials I and II we will use the same mesh density for the case that follow.

IV.5. **Strip Case C, material II**

We recalculated the response of Case C using the new stress–strain response with all geometric variables remaining the same. The force-elongation response is shown in Fig. 24(a), and a sequence of black-and-white deformed configurations is shown in Fig. 25. In this case we also include (Fig. 24(b)) a plot of the moment \((M)\) acting at the end(s) of the strip \((x = \pm L/2)\) as a function of the applied end-displacement \(\delta\). The moment is normalized by the “yield” moment \(M_0 = \sigma_0 \delta^{\frac{d}{2}}\) where \(\sigma_0\) is the “yield” stress, i.e. the local maximum of the stress–strain response, in Fig. 23(b).

The initiation of the instability is now a more distinct event associated with a sharper and higher amplitude load drop. The result is again inclined bands similar to those seen in the experiments. The fronts, which for this material have a steeper gradient, propagate in

![Fig. 19. Lateral displacement of strip axis of Case B (material I) as a function of \(\delta\).](image-url)
Fig. 20. Force–displacement response for Case C (material I).

Fig. 21. Sequence of black-and-white strain contours for Case C (material I) at 2% cutoff strain.
both directions causing broadening of the bands. As before, the inclined bands cause kinking of the strip which is seen in the $v(x) - \delta$ plot in Fig. 26. Between configurations 1–5 the local lateral deflection grows, and this increases the local moment acting on the bent strip (Fig. 24(b)). By configuration 6, the outward propagating fronts start to interact with the ends of the strip while the other two fronts continue to propagate towards the center of the specimen. By configuration 7, the deformation at the ends of the strip is nearly complete (note the drops in the net axial load and end moment). When the triangular regions in the corners of the strip are deformed (in the neighbourhood of configuration 8), the magnitude of the moment in the strip increases once more (Fig. 24(b)) because the lengths of the edges of the deformed regions are significantly different from each other. This, forces a new nucleation of a finger along the alternate characteristic. The new deformation pattern straightens the specimen and relieves the moment. In Fig. 26 we see that by configuration 9 the strip is nearly straight again.

In contrast to the material I strip, in the present case the fronts propagate by progressive initiation of finger-like wedges from the edges of the strip (see configurations 8–12). Initiation of such fingers of deformed material is accompanied by small load peaks and end moment undulations which are registered in Fig. 24(a) and (b), respectively. The net result of the successive initiation of such fingers is that the average stress in the strip is somewhat higher (by $\sim 10$ MPa) than the Maxwell value (which is achieved during steady state propagation of self-similar fronts). The calculation was interrupted when the fronts reached the course mesh in the inner portion of the specimen.

We note that in our experiments the ends of the NiTi strips were clamped between two plates. This is a complex boundary condition which we did not attempt to model. The
idealized end condition used in the simulation is probably the main reason for the difference in the calculated deformation patterns after configuration 8 in Fig. 25 and the patterns seen in Fig. 1. However, propagation of fronts in this criss-cross manner has been observed in some other experiments on long strips pulled at slow end-displacement rates (e.g. Fig. 4(c)).

In summary, we see that the use of material II has had the expected effect of sustaining the propagation of the two inclined bands of deformed material for a much longer portion of the deformation history than for material I. Because of the position at which the bands were initiated, the outer fronts eventually run into the ends of the strips and that caused the switch to the more symmetric criss-cross front pattern of propagation.

IV.6. Strip Case D, material II

The last case that we will consider is a simulation of a tension test on a short (14 × 4 mm) strip like the one shown in Fig. 5 using material II. The length of the strip is 3.5 a and the imperfection is placed at one of the corners to initiate the instability in the same neighbourhood as that of the experiment in Fig. 5. The results are summarized in Figs 27–31. The axial force and moment (at x = 0) end displacement responses are shown in Fig. 27(a) and (b), respectively. A sequence of deformed configurations in color are presented in Fig. 28 and the corresponding black-and-white versions are shown in Fig. 29 (cutoff strain 2%).

The events are quite different from those observed for material I either in Fig. 12 or in Fig. 18. Instability initiates from the corner imperfection and takes the form of an inclined band (configuration 1). This causes the specimen to kink (see ν(x) − δ plot in Fig. 30) and a bending moment to develop (Fig. 27(b)). Soon, the triangular region of relatively undeformed material at the lower end starts to deform which temporarily relieves the moment. As the deformation evolves, a finger-like island of undeformed material, visible
Fig. 24. (a) Force–displacement response for Case C (material II). (b) Corresponding end moment-displacement response.

in configurations 4 and 5, is left behind. In the neighbourhood of configuration 5, the amount of deformed material on the lower LHS is more than on the lower RHS and the moment is driven in the opposite sense. In order to relieve this moment, instability initiates at a new site on the RHS edge of the strip just ahead of the front along the alternate characteristic (see configuration 6).

The new initiation temporarily makes the deformed pattern more symmetric, which results in a reduction in the lateral deflection and the moment. However, because of its finger-like nature, the new deformation pattern leaves behind a wedge-like island of undeformed material (configurations 6 and 7). In addition, the new band results in some kinking which grows as the band widens. This local kinking causes an asymmetry and an
Fig. 25. Sequence of black-and-white strain contours for Case C (material II) at 2% cutoff strain.

Fig. 26. Lateral displacement of strip axis of Case C (material II) as a function of $\delta$. 
increase in moment. Before the band can widen very far, the moment becomes too severe and is relieved by another finger-like initiation on the LHS edge of the strip (configuration 7). As before, a wedge-like island of undeformed material is left behind. In addition, the opposite orientation of the new band causes some kinking in the opposite direction. This eventually results (configuration 8) in the initiation of a new finger on the RHS of the strip and a repetition of this cycle of events.

Indeed, the rest of the strip is deformed through successive initiation of finger-like features which alternately form along the two longitudinal edges of the short strip. Each cycle is responsible for the undulations in the moment (Fig. 27(b)) and in the net axial force (Fig. 27(a)). Seven pairs of fingers are seen to develop in the specimen before the front nears the upper edge. This successive initiation of deformation and the erasure of islands of undeformed material produced by the finger-like features, do not allow the deformation history to settle to a steady-state. As a result, the average nominal stress recorded in Fig. 27(a) is 398 MPa which is higher than the Maxwell level of 389 MPa.

The complex clamped conditions used in the experiments and the presence of small geometric imperfections in the NiTi strips, do not allow an exact, one-to-one comparison between the measured and calculated deformation features. However, from an overall comparison of the black-and-white images in Fig. 29 and those in Fig. 5 we can conclude without hesitation that the calculated results exhibit all the features seen in the experiment. The qualitative similarity of finger-like features and the wedges of undeformed material left in their wake is quite striking. Interestingly, the period of the finger cycles in the two sets of results and the amplitudes of the net force undulations were also very similar.

Incidentally, in the case of material II the sensitivity of deformation patterns in the black-white representations to the cut-off strain is much less than for material I. This is illustrated in Fig. 31 where configuration 11 is displayed as a function of cutoff strain. Although the finger-like features become stronger as the cutoff strain is increased, the overall qualitative pattern does not change. The unstable branch of material II makes intermediate strain states unfavorable, and therefore, causes the transition fronts to be steeper than those of material I. By contrast, in the case of material I, the V-shaped propagation fronts have a "bowtie" zone of material at these intermediate strains (Figs 12 and 18). The reduced initiation peak allowed the existence of these wide transition zones. Finally, we also note that despite the unstable nature of material II, the deformation patterns reported here were not altered in any significant manner by refining the mesh size further (Shaw, 1997).

V. SUMMARY AND CONCLUSIONS

Displacement controlled uniaxial tension tests on NiTi strips have shown that $A \Rightarrow M$ phase transformations result in inhomogeneous deformations. Transformation nucleates in narrow bands inclined at 55° to the axis of loading. Under isothermal conditions, subsequent transformation takes place at a well defined stress plateau which is lower than the nucleation stress (for $A \rightarrow M$). During the stress plateau, phase fronts propagate along the length of the specimen. In longer specimens, the fronts tend to be relatively sharp, transition zones in strain inclined at 50 to 60° to the direction of loading. Shear deformation associated with such inclined bands of transformed material causes kinking of the strip. The amount of kinking depends on the aspect ratio of the strip, on the boundary
conditions used, on the axial position of the front, on the transformation strain and on the axial stress. In some cases the extent of kinking could not be sustained, and this caused either a switch of the direction of the inclined band to the alternate characteristic or propagation via a criss-cross pattern of finger-like features. The successive nucleation of such fingers results in small undulations in the net axial stress. The criss-cross mode of front propagation was more prominent in shorter specimens in which strip kinking is more difficult.

Interestingly, several of these characteristics of inhomogeneous deformations associated with phase transformations have also been observed during the propagation of Lüders bands in uniaxial tension tests on fine-grained steel strips and wires. In this dislocation
governed phenomenon the transformation strain is replaced by the Lüders strain. The macroscopic stress-elongation response, the sharp bands of deformed material and their inclinations, specimen kinking, and the way two converging fronts interact are indeed very similar to those described above for NiTi strips. These similarities at the macroscopic level, despite the fundamental differences of the two phenomena at the micro-level, prompted us to conclude that macroscopic geometric effects and the underlying mechanism of continuum strain localization must play an important role in the observed behavior. Motivated by this we used finitely deforming $J_2$-type continuum plasticity to conduct FE simulations of several strips in tension.

Two somewhat different trilinear fits of the NiTi force-elongation response were employed. The nominal stress–strain versions of both fits had a middle branch with a negative slope. Material I had a monotonically increasing true stress–strain response, whereas the middle branch of the true stress–strain response for material II had a negative slope which better represented the experimental data. Simulated uniaxial tension tests produced inhomogeneous deformations with features similar to those of the experiments for both fits. These can be summarized as follows:

- Instability initiated in the form of a narrow band with a 55° inclination to the loading direction for both material models.
- Initially, the band broadened, causing kinking of the strip axis. The resultant moments often caused a switch to alternate deformation modes which relieved the moments. The time of this switch and the choice of new mode was shown to depend

![Image](image_url)

Fig. 29. Sequence of black and white strain contours of Case D for 2% cutoff strain.
Fig. 30. Lateral displacement of strip axis of Case D (material II) as a function of $\delta$.

Fig. 31. Black-and-white strain contours for configuration 11 of Case D at different cutoff strains.
on the size of the nucleation peak (material I small, material II larger), on the length of the strip, and on the boundary conditions adopted.

- In material I strips the inclined bands were short lived. Using finger-like deformation features the inclined bands reverted to V-shaped ones, which by their symmetry straightened the strip axis. Associated with such fronts were relatively large zones of material in the intermediate (unstable) strain range. Steady-state propagation of the V-shaped fronts took place near the Maxwell stress.

- In material II strips the inclined bands were longer lived. The higher nucleation stress of this material made the fronts much sharper in order to limit the amount of material in the unstable region. As a result, V-shaped fronts could not be sustained. Instead, deformation eventually propagated by the successive nucleation of fingers along alternating characteristics. This resulted in corresponding undulations in the average axial stress and in the moment. The initiation of each finger initially relieved the moment but because of its unsymmetric nature soon drove the moment in the opposite direction as it widened, which in turn triggered a new nucleation on the other side of the strip. This continued until the whole strip was deformed. Since steady-state propagation of a self-similar front never materialized, the average axial stress was somewhat higher than the Maxwell stress.

- Overall, the events of the simulations with material II were very similar to events observed in the NiTi strips. Since the difference between the upper and lower yield stress of steels is significantly larger than the nucleation peak of NiTi, the inclined bands remain the preferred mode for Lüders bands propagation.

- Solutions from material II exhibited some modest sensitivity to mesh. We chose to use the same mesh which produced converged solutions for material I. Unlike other problems of localization due to material instability, the presence of the third stable branch in the stress-strain response (after a strain of $\sim5\%$) had an overall stabilizing effect on the solution since it limits the extent of unstable behavior. As a result, the observed mesh sensitivity affected mainly the shape of the transition zones between the relatively undeformed and the deformed regions of the strip.

- For both material models when converging fronts coalesced the overall force in the specimen exhibited small dips similar to those observed in tension tests on NiTi and steel strips.

The results of the numerical simulations confirm that the evolution of events seen in experiments on SMAs and mild steels is strongly influenced by overall geometric (structural) effects. Furthermore, the success of the simple continuum constitutive model strongly suggests that continuum level events remain dominant players in such fine grained materials irrespective of the micromechanical root cause of the instability.

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